your name(s)______________________________

Physics 321 Quiz #5 – Friday Feb. 8

Work in groups of 3, open notes/book/internet/mouth

A particle of mass m and charge q is confined to a harmonic oscillator with spring constant k. There is a very small damping force, $-bv$ with $b \to 0^+$. An oscillating electric field is applied for a long time, much longer then the relaxation time $2m/b$. The electric field has a square wave form and a periodicity τ, $E(t+\tau) = E(\tau)$,

$$E(t) = \begin{cases} 0, & -\tau/2 < t < 0 \\ E_0, & 0 < t < \tau/2 \end{cases}$$

1. (5 pts) If you represent the electric field described as a Fourier transform,

$$E(t) = \frac{f_0}{2} + \sum_{n>0} f_n \cos(n\omega t) + g_n \sin(n\omega t), \quad \omega = 2\pi/\tau,$$

   which f_n and g_n are non-zero? f_0 & g_1, g_3, g_5 - - - . .

2. (10 pts) Find the Fourier coefficients.
3. (5 pts) Find $x(t)$, expressing answer as a sum. You can apply Eq.s (3.31) and (3.32) from the text.
4. (5 pts) For what values of τ is the amplitude of the motion singular?

(2)

$$g_n = \frac{4}{\tau}\int_0^{\tau/2} E_0 \sin(n\omega t), \quad \omega = 2\pi/\tau$$

$$= \frac{4E_0}{\omega\tau \cdot n}\left[-\cos(n\pi\tau) + 1\right]$$

$$= \frac{2E_0}{\pi n} \quad \text{for } n = 1, 3, 5, 7 \cdots$$

$$= 0 \quad \text{otherwise}$$

$$\langle E \rangle = E_0/2$$

$f_0 = E_0$, so

$f_{n>0} = 0$

(3) $$x(t) = \frac{qE_0}{2k} + \sum_n \frac{q\, g_n}{m} \frac{\sin(n\omega t)}{\omega_0^2 - \omega^2 \cdot n^2}$$

(4) $x \to \infty$ for $n\omega = \omega_0 = \frac{2\pi n}{\tau}$, $\tau = 2\pi n\sqrt{\frac{m}{k}}$