Physics 321 Quiz #7 - Friday, Feb. 22

Conisder a spherical asteroid made of rock with density $\rho_a = 2$ g/cm^3 and radius a. It is in a circular orbit of radius r about Saturn (mass M_s). A pebble of mass δm lies on the surface of the asteroid, either at a point facing, or opposite the direction of Saturn.

1. In terms of G, M_S, r, a and δm, express the tidal force on the pebble. The tidal force is the difference between the gravitational force from Saturn acting on the pebble, and the force the pebble would experience if it were at the center of the asteroid. Keep only the first term when expanding as a function of a.

2. In terms of G, a δm and ρ_a, what is the gravitational force acting on the pebble due to the gravitational attraction to the asteroid.

3. Equate the two forces to find the radius r at which the tidal force rips apart the asteroid. Saturn's mass is $M_S = 5.68 \times 10^{26}$ kg, and $G = 6.67 \times 10^{-11}$ Nm^2/kg^2. Compare you answer to the radius of Saturn, $R_S = 5.82 \times 10^4$ km.

4. What would happen if the asteroid were made of ice instead of rock?

5. Express your answer for (3) in terms of R_S, ρ_a and the density of Saturn ρ_S. Note that the density of Saturn is 0.9 g/cm^3. It would float in your bathtub, if your bathtub were large enough. The average density of Jupiter is 1.3 g/cm^3, and the average density of Earth is 5.5 g/cm^3.

6. If a small moon and planet slowly spiral toward each other, and if the moon and planet have the same density, will the moon be torn apart by tidal forces before reaching the surface of the planet?

1. $F_{tidal} = \dfrac{2 G M_s \delta m}{r^3} a$

2. $F_a = G \dfrac{4\pi a^3 \rho_a \delta m}{3 \quad a^2} = \dfrac{4\pi}{3} G \rho_a \delta m \, a$

3. $\dfrac{2 M_s}{r^3_{crit}} = \dfrac{4\pi}{3} \rho_a$, $r_{crit} = \left[\dfrac{3 M_s}{2\pi \rho_a} \right]^{1/3}$

4. r_{crit} would increase by $(2^{1/3})$
because $\rho_{ice} \cong 1.0$

5. $r_{crit} = \left(\dfrac{2 \rho_s}{\rho_a} \right)^{1/3} R_s$

6. $r_{crit} = 2^{1/3} R$, so the moon rips apart BEFORE reaching planet