Group I problems, Dec. 2, 2016

DO NOT WRITE YOUR NAME OR STUDENT NUMBER ON ANY SHEET!

FUN FACTS TO KNOW AND TELL

$$\int_0^\infty dx\ \frac{x^{n-1}}{e^x-1} = \Gamma(n)\zeta(n), \qquad \int_0^\infty dx\ \frac{x^{n-1}}{e^x+1} = \Gamma(n)\zeta(n)\left[1-(1/2)^{n-1}\right],$$

$$\zeta(n) \equiv \sum_{m=1}^{\infty} m^{-n}, \quad \Gamma(n) \equiv (n-1)!,$$

$$\zeta(3/2) = 2.612375..., \quad \zeta(2) = \frac{\pi^2}{6}, \quad \zeta(3) = 1.20205..., \quad \zeta(4) = \frac{\pi^4}{90},$$

$$\int_{-\infty}^{\infty} dx\ e^{-x^2/2} = \sqrt{2\pi}, \qquad \int_0^\infty dx\ x^n e^{-x} = n!$$

Problem from Sections 1.10-1.11

——Jacob Calcutt and Aaron Magilligan——

Starting with the expression,

$$TdS = dE + PdV - \mu dN \tag{1}$$

Derive the following Maxwell Relation:

$$\frac{\partial \mu}{\partial P} |_{S,N} = \frac{\partial V}{\partial N} |_{S,P} \tag{2}$$

Kristen Dage and Devyn Rysewyk

Consider the equation of state,

$$P = \rho T e^{\rho/\rho_o} - a\frac{\rho^2}{\rho_o}$$

Derive an expression for the energy per particle, E/N, as a function of the temperature, T, and the density, ρ, and the parameters ρ_o and a. Start your derivation with the Maxwell relation,

$$\left.\frac{\partial E}{\partial V}\right|_{\beta,N} = -\left.\frac{\partial(\beta P)}{\partial \beta}\right|_{V,N}$$

Joe Lundeen and Sara Ayoub

Review Question

Consider a boson (spin 1) confined to a system with two energy levels (0 and ε) independent of the spin.

(a) Find the entropy S in both the $T=0$ and $T=\infty$ limits

(b) Repeat (a) assuming the particle is a fermion (spin ½) instead.

PHY 831: Final Review Question

Mengzhi Chen, Jacob Morrison

1. Consider classical non-relativistic particles acting under a spherically symmetric potential,

$$V(\vec{r}) = V_0 e^{\vec{r}^2/\lambda}.$$

Show that

$$\left\langle \frac{2\vec{r}^2}{\lambda} V(\vec{r}) \right\rangle = 3T.$$

Daniel Paz and Dayah Chrisman

Find the energy density of a gas of massless electrons in three dimensions. Let the temperature be T and $\mu = 0$

3.5-3.6 Quiz Question

Rachel Salmon and Brooke Edgar

December 2, 2016

1 Question

Considering the virial expansion (Equation 1). For a gas of non-interacting bosons, is A_2 positive, negative, or zero?

$$P = \rho T + \rho T \sum_{n=2} A_n \left(\frac{\rho}{\rho_0}\right)^{n-1} \tag{1}$$

2 Question

Calculate A_2 in Equation 1 for a non-relativistic gas. Assume an S-wave where $\delta_{l=0} = -\frac{ap}{\hbar}$.

Jaclyn Schmitt
Austin Edmister
Sections 2.6 & 2.7

a. Grand Canonical vs. Canonical (Qualitative)
   Under what conditions can you approximate a canonical ensemble as a grand canonical ensemble?

b. Gibb's Paradox (Qualitative)
   You have two adjacent boxes of volume V: One box with N_a "a" particles and the other with N_b "b" particles (Figure 1). We know that when the partition is lifted, the change in entropy per particle is $\Delta(S/N) = \ln 2$, where N is the total number of particles. If you add N_b "b" particles to side "a" (Figure 2), is the new change in entropy per particle greater than, less than, or equal to the previous result?

Figure 1. $\Delta(S/N) = \ln 2$

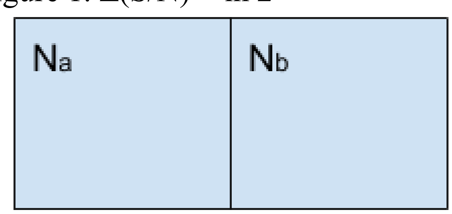
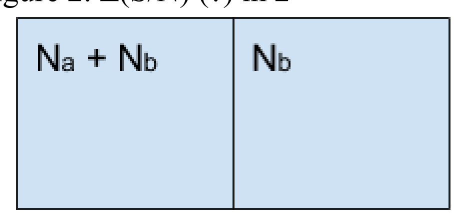


Figure 2. $\Delta(S/N)$ (?) $\ln 2$



Quiz Question: Sections 1.5-1.9

Tamas Budner, Alex Madden

November 30, 2016

The Helmholtz free energy is defined

$$F = E - TS.$$

Using this and the fundamental thermodynamic relation, show that

$$\mu = \frac{\partial F}{\partial Q}\bigg|_{V,T} \qquad \text{and} \qquad \beta = \frac{\partial S}{\partial E}\bigg|_{V,Q}.$$