Physics 831 Practice Quiz #4 - Wednesday, Nov. 28

YOUR NAME:____________________________

FUN FACTS TO KNOW AND TELL

$$\begin{aligned}
\int_0^\infty dx\,\frac{x^{n-1}}{e^x-1} &= \Gamma(n)\zeta(n), \qquad \int_0^\infty dx\,\frac{x^{n-1}}{e^x+1} = \Gamma(n)\zeta(n)\left[1-(1/2)^{n-1}\right],\\
\zeta(n) &\equiv \sum_{m=1}^{\infty} m^{-n}, \quad \Gamma(n) \equiv (n-1)!,\\
\zeta(3/2) &= 2.612375..., \quad \zeta(2)=\frac{\pi^2}{6}, \quad \zeta(3)=1.20205..., \quad \zeta(4)=\frac{\pi^4}{90},\\
&\int_{-\infty}^{\infty} dx\, e^{-x^2/2} = \sqrt{2\pi}, \qquad \int_0^\infty dx\, x^n e^{-x} = n!
\end{aligned}$$

1. (2 pts each) Consider a single electron that can be either spin $\uparrow$ or $\downarrow$, with the two energies being ϵ or $-\epsilon$.

   (a) What is the average energy when $T = 0$?

   (b) What is the average energy when $T \to \infty$?

   (c) What is the specific heat when $T = 0$?

   (d) What is the specific heat when $T \to \infty$?

   (e) What is the entropy when $T = 0$?

   (f) What is the entropy when $T \to \infty$?

2. (10 pts) Beginning with the fundamental thermodynamic relation, and the definition of the specific heat,

$$TdS = dE + PdV - \mu dN, \quad C_V = T \left.\frac{\partial S}{\partial T}\right|_{N,V},$$

derive the relation:

$$-\frac{1}{T}\left.\frac{\partial C_V}{\partial N}\right|_{T,V} = \left.\frac{\partial^2 \mu}{\partial T^2}\right|_{V,T}$$

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3. (10 pts) Consider a particle moving in a potential well:

$$V(x) = -A\ln(x/x_0) + Bx, \quad \text{where } (x_0 > 0, A > 0, B > 0),$$

which confines a particle $0 < x < \infty$.
Find $\langle x \rangle$ as a function of A, B, x_0 and the temperature T.

4. Massless electrons ($\epsilon = pc$, and can be either spin up or spin down) move in **two dimensions** and equilibrate to a temperature T.

   (a) (5pts) For $T = 0$ and chemical potential μ, find the density (number per area) ρ in terms of μ.

   (b) (5pts) What is $D(\epsilon)$, the density of single particle states?

   (c) (5pts) To order T^2, find the change in the chemical potential $\delta\mu$ necessary to maintain constant density. Express answer in terms of μ and T.