1. Consider a two-dimensional Fermi gas of spin-1/2 particles of mass \( m \) at low temperature \( T \) and fixed density (number per area) \( \rho \). Here the chemical potential will be measured relative to \( p = 0 \), i.e.,
\[
      f = \frac{e^{-\beta(p^2/2m-\mu)}}{1 + e^{-\beta(p^2/2m-\mu)}}
\]
(a) What is the single-particle density of states per volume \( D(\epsilon)/V \) at \( \epsilon = \mu \)?
(b) What is the net kinetic energy per particle at \( T = 0 \)? Give answer in terms of \( \epsilon_f = \mu \) and other given constants.
(c) What is the increase in the excitation energy per particle to order \( T^2 \)? Give answer in terms of \( \rho \) and \( D/V \).

2. Consider a Hamiltonian for a particle moving in one dimension,
\[
      H = \sqrt{m^2 + p^2} - A \ln(x/x_0) + Bx, \quad A > 0, B > 0.
\]
which confines the particle to \( 0 < x < \infty \). Using the generalized equipartition theorem, or the virial theorem, find the average value of \( \langle x \rangle \) as a function of \( T \).