Physics 831 Quiz #11 - Wednesday, Nov. 16

YOUR NAME:______________________________

1. Consider the following form for the free-energy density for a spin system where the σ can be between -1 and +1:

$$\begin{aligned} f(\sigma, T) &= \rho_0 \left\{ \mathcal{V}(\sigma, T) + \frac{\kappa}{2}(\nabla\sigma)^2 \right\}, \\ \mathcal{V}(\sigma, T) &= -\frac{1}{2}J\sigma^2 + T\frac{(1+\sigma)}{2}\ln\left(\frac{1+\sigma}{2}\right) + T\frac{(1-\sigma)}{2}\ln\left(\frac{1-\sigma}{2}\right). \end{aligned}$$

Fill in either *increase* or *decrease* for the following statements concerning the surface energy between a region where the spins have minimized f with $\sigma > 0$ and a separate region where $\sigma < 0$.

(a) The surface energy will ____________ if κ is increased.

(b) The surface energy will ____________ if J is increased.

(c) The surface energy will ____________ if T is increased (but kept less than T_c).

2. Beginning with the definition of the partition function,

$$Z = \mathrm{Tr}\ e^{-\beta H_{\rm int} + \beta\mu \int d^3r B m(\mathbf{r})},$$

where $H_{\rm int}$ is the internal Hamiltonian, m is the spin density and B is an external field, show that the correlation function,

$$\Gamma(r) \equiv \langle m(0)m(r)\rangle - \langle m\rangle^2,$$

satisfies the relation:

$$\int d^3r\Gamma(r) = \frac{1}{(\beta\mu)^2}\frac{d^2(\ln Z/V)}{dB^2},$$

where V is the volume.