

Physics 831 Quiz #8 - Friday, Oct. 26

YOUR NAME: _____

1. Consider a two-dimensional array of N coupled three-dimensional harmonic oscillators. The array supports both a longitudinal and two transverse sound modes.

(a) At low temperature, the specific heat per oscillator, $(dE/dT)/N$, can be expressed as:

$$C/N = \alpha T^n.$$

What is the power n ?

(b) What is the specific heat per oscillator at high temperature?

2. Consider an independent spin whose values are either $\sigma_i = +1$ or -1 . Beginning with the definition of entropy,

$$S = - \sum_{\ell} p_{\ell} \ln p_{\ell},$$

where p_{ℓ} is the probability of a given level ℓ being occupied:

(a) Find S in terms of $\langle \sigma \rangle$.

(b) Assuming the spins have an interaction energy per spin,

$$E/N = -\frac{J}{2} \langle \sigma \rangle^2,$$

Derive an expression (perhaps transcendental) for $\langle \sigma \rangle$ that minimizes the free energy, $F = E - TS$.

(c) How does your expression compare to the one given in the notes, $\langle \sigma \rangle = \tanh(\beta J \langle \sigma \rangle)$. If different, explain what physical assumptions are driving the difference.