1. The diagram represents a perturbative calculation of the partition function.

(a) Consider the connected diagram involving \( p_a \to p_d \). When calculating the pressure, this diagram contributes to order \( n = \_\_\_\_\_\_ \) in perturbation theory.

(b) When performing a virial expansion (see expansion below), the lowest \( m \) for which this contributes to \( A_m \) is \_\_\_\_\_\_\_\_\_.

2. Consider a virial expansion for a non-relativistic ONE-dimensional gas of spin-zero bosons of mass \( m \) at temperature \( T \),

\[
\frac{P}{\rho T} = 1 + \sum_{m=2}^{\infty} A_m \left( \frac{\rho}{\rho_0} \right)^{m-1}, \quad \rho_0 \equiv \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} dp \ e^{-p^2/2mT}.
\]

Ignoring interactions between the particles, calculate \( A_2 \). Here \( \rho \) is the number per unit length. Begin with the expression for the one-dimensional “pressure”, and the density

\[
\frac{PL}{T} = \ln Z = \frac{L}{2\pi\hbar} \int_{-\infty}^{\infty} dp \ \ln \left( \frac{1}{1 - e^{-\beta(p^2/2m-\mu)}} \right);
\]

\[
\rho = \frac{1}{L} \frac{\partial}{\partial \beta \mu} \ln Z.
\]

3. Consider the state:

\[
| \eta \rangle = e^{(\eta a^\dagger - \eta^* a)} | 0 \rangle.
\]

Find the overlap \( \langle 0 | \eta \rangle \).

Hint: you may want to use the Campbell-Baker-Hausdorff lemma – If operators \( A \) and \( B \) commute to a number,

\[
e^{A+B} = e^A e^B e^{-[A,B]/2}
\]