1. Consider a gas of non-relativistic one-dimensional zero-temperature fermions of mass \( m \), filling up all states with momenta, \(-p_f < p < p_f\). The system is also confined to a region, \(-L < x < L\). This gives a phase space density

\[
f(p, x, t < 0) = \Theta(p + p_f)\Theta(p - p)\Theta(x + L)\Theta(L - x),
\]

where \( \Theta \) is the step function. At \( t = 0 \), the boundaries disappear suddenly and the particles move on toward oblivion without collisions.

(a) Find \( f(p, x, t) \) for \( t > 0 \).

(b) What is the net entropy at \( t = 0 \)?

(c) What is the net entropy as a function of \( t \), for \( t > 0 \).

2. (Extra Credit) Consider an infinitely deep one-dimensional square-well of width \( L = 1.0 \) nm. The well traps an electron and thermalizes at a temperature of 1.0 nano-Kelvin. The probability that the electron is in the ground state is:

(a) exactly, or very nearly, zero

(b) exactly, or very nearly, 1

(c) more than 1%, less than 99%

3. (Extra Credit) Now consider a single hydrogen atom in a very large box (approaches infinity) at the same temperature, 1.0 nano-Kelvin. What is the probability the electron occupies the ground state (-13.6 eV binding energy) of the hydrogen atom.

(a) exactly, or very nearly, zero

(b) exactly, or very nearly, 1

(c) more than 1%, less than 99%

Fun Facts to know and tell:

\[
S = \frac{1}{(2\pi\hbar)} \int dx \, dp \ [\pm(1 \pm f) \ln(1 \pm f) - f \ln(f)], \quad \text{for bosons/fermions.}
\]

1 eV=1.15 \times 10^{14} \text{ nano-Kelvin.}