Physics 831 Quiz #6 - Friday, Oct. 10

1. Consider a gas of non-relativistic one-dimensional zero-temperature fermions of mass m, filling up all states with momenta, $-p_f < p < p_f$. The system is also confined to a region, $-L < x < L$. This gives a phase space density

$$f(p, x, t < 0) = \Theta(p + p_f)\Theta(p_f - p)\Theta(x + L)\Theta(L - x),$$

where Θ is the step function. At $t = 0$, the boundaries disappear suddenly and the particles move on toward oblivion without collisions.

   (a) Find $f(p, x, t)$ for $t > 0$.

   (b) What is the net entropy at $t = 0$?

   (c) What is the net entropy as a function of t, for $t > 0$.

2. (Extra Credit) Consider an infinitely deep one-dimensional square-well of width $L = 1.0$ nm. The well traps an electron and thermalizes at a temperature of 1.0 nano-Kelvin. The probability that the electron is in the ground state is:

   (a) exactly, or very nearly, zero

   (b) exactly, or very nearly, 1

   (c) more than 1%, less than 99%

3. (Extra Credit) Now consider a single hydrogen atom in a very large box (approaches infinity) at the same temperature, 1.0 nano-Kelvin. What is the probability the electron occupies the ground state (-13.6 eV binding energy) of the hydrogen atom.

   (a) exactly, or very nearly, zero

   (b) exactly, or very nearly, 1

   (c) more than 1%, less than 99%

Fun Facts to know and tell:

$$S = \frac{1}{(2\pi\hbar)} \int dx\ dp\ \left[\pm(1 \pm f)\ln(1 \pm f) - f\ln(f)\right], \qquad \text{for bosons/fermions.}$$

1 eV=1.15×10^{14} nano-Kelvin.
