

Physics 831 Quiz #9 - Wednesday, Nov. 5

YOUR NAME: _____

Consider the 1-d ising model on $N \rightarrow \infty$ spins, where each spin can have $\sigma_i = \pm 1$ and the Hamiltonian is:

$$H = - \sum_i J \sigma_i \sigma_{i+1} - \mu B \sigma_i,$$

and the partition function is given by:

$$\ln Z = N \ln(\lambda), \quad \lambda = e^{\beta J} \cosh(\beta \mu B) + \sqrt{e^{2\beta J} \sinh^2(\beta \mu B) + e^{-2\beta J}}.$$

1. Consider the case where $J = 0$.

- (a) What is the average spin per $\langle \sigma \rangle$ as a function of μB and T ?
- (b) What is the fluctuation of the net spin (assume N spins)? The fluctuation is defined as:

$$F = \frac{1}{N} \left\langle \left(\sum_i \sigma_i \right)^2 \right\rangle - \frac{1}{N} \left\langle \sum_i \sigma_i \right\rangle^2$$

2. Consider the case where $J \neq 0$ and $\mu B = 0$. Derive the probability that the first n spins are +1 while the n^{th} spin is -1.