1. A molecule of mass $m$ has internal excitations consistent with that of a **TWO-DIMENSIONAL** harmonic oscillator with tightly packed levels, $\hbar \omega << T$. Initially, a gas of such molecules is at temperature $T_i$ before expanding and cooling adiabatically to a temperature $T_f$. Neglect quantum degeneracy of the momentum states for the following questions. (HINT: A two-dimensional oscillator behaves like two independent one-dimensional oscillators.)

(a) Find the average energy per particle in terms of the temperature $T$, the mass $m$ and $\hbar \omega$.

(b) Derive an expression for the initial entropy per particle in terms of $m$, $T_i$, $\hbar \omega$ and the initial density $\rho_i$.

(c) After adiabatically cooling to temperature $T_f$, find the density $\rho_f$. Give answer in terms of $T_i$, $T_f$ and $\rho_i$. 
2. (Extra Credit) Consider a fluid with an ideal gas equation of state, \( P = \rho T \), and a mass density \( \rho_m = m \rho \). The energy density is that of a non-interacting gas, \( \epsilon = (3/2)\rho T \). At time \( t = 0 \), the temperature is uniform, \( T = T_0 \), and the collective velocity is zero everywhere, but the density varies exponentially (as far as the eye can see),

\[
\rho(x, t = 0) = \rho_0 e^{-x/\lambda}.
\]

Solve for the evolution of the density \( \rho(x, t) \), the collective velocity \( v(x, t) \), and the temperature \( T(x, t) \), by solving the equations:

\[
(\partial_t + v \partial_x)v(x,t) = -\frac{\partial_x P(x,t)}{m \rho(x,t)},
\]
\[
(\partial_t + v \partial_x)\rho(x,t) = -\rho(x,t) \partial_x v(x,t),
\]
\[
(\partial_t + v \partial_x)\epsilon(x,t) = -[P(x,t) + \epsilon(x,t)] \partial_x v.
\]

Hint: Use your intuition and assume SIMPLE forms for the time and spatial dependence of \( v, \rho, T, \cdots \).