Physics 831 Quiz #4 - Monday, Nov. 2

1. Consider a gas of non-relativistic one-dimensional zero-temperature spin-1/2 fermions of mass m, filling up all states with momenta, $-p_f < p < p_f$. The system is also confined to a region, $-L < x < L$. This gives a phase space density,

$$f(p, x, t < 0) = \Theta(p + p_f)\Theta(p_f - p)\Theta(x + L)\Theta(L - x),$$

where Θ is the step function. At $t = 0$, the boundaries disappear suddenly and the particles move on toward oblivion without collisions.

   (a) Find $f(p, x, t)$ for $t > 0$.

   (b) What is the density at $x = 0$ as a function of time for $t > 0$?

   (c) What is the net entropy at $t = 0$?

   (d) What is the net entropy as a function of t, for $t > 0$.

2. A point source of perfume is responsible for N molecules at position $x = x_0$ at $t = 0$ (assume $x_0 > 0$). The molecules diffuse in one dimension according to the diffusion constant D as defined by the diffusion equation,

$$\frac{\partial \rho(x,t)}{\partial t} = D \frac{\partial^2 \rho(x,t)}{\partial x^2}.$$

   (a) Derive an expression for the density as a function of time.

   (b) Assume that at $x = 0$, there is a wall that absorbs all molecules that contact the wall. What is the boundary condition for $\rho(x = 0, t)$? Re-derive $\rho(x, t)$.

   (c) Assume that at $x = 0$ there is a wall that reflects all molecules that contact the wall. What is the boundary condition for $\rho(x = 0, t)$? Re-derive $\rho(x, t)$.