1. The diagram represents a perturbative calculation of the partition function.

   (a) Consider the connected diagram involving $p_a \to p_d$. When calculating the pressure, this diagram contributes to order $n =$ _______ in perturbation theory.

   (b) When performing a virial expansion (see expansion below), the lowest $m$ for which this contributes to $A_m$ is _______.

2. Consider the state:

   $$|\eta\rangle = e^{(\eta a^\dagger - \eta^* a)}|0\rangle.$$ 

   Find the overlap $\langle 1|\eta \rangle$.

   Hint: you may want to use the Campbell-Baker-Hausdorff lemma – If operators $A$ and $B$ commute to a number,

   $$e^{A+B} = e^A e^B e^{-[A,B]/2}$$
3. Consider a virial expansion for a non-relativistic two-dimensional gas of spin-1/2 Fermions of mass $m$ at temperature $T$,

$$\frac{P}{\rho T} = 1 + \sum_{m=2}^{\infty} A_m \left( \frac{\rho}{\rho_0} \right)^{m-1}, \quad \rho_0 \equiv \frac{1}{(2\pi\hbar)^2} \int_{-\infty}^{\infty} 2\pi p \, dp \, e^{-p^2/(2mT)}.$$

Ignoring interactions between the particles, calculate $A_2$. Here $\rho$ is the number per unit length. Begin with the expression for the two-dimensional “pressure”, and the density

$$\frac{PA}{T} = \ln Z = \frac{2A}{(2\pi\hbar)^2} \int_{-\infty}^{\infty} 2\pi p \, dp \, \ln \left( 1 + e^{-\beta(p^2/2m-\mu)} \right),$$

$$\rho = \frac{1}{A} \frac{\partial}{\partial \beta \mu} \ln Z.$$