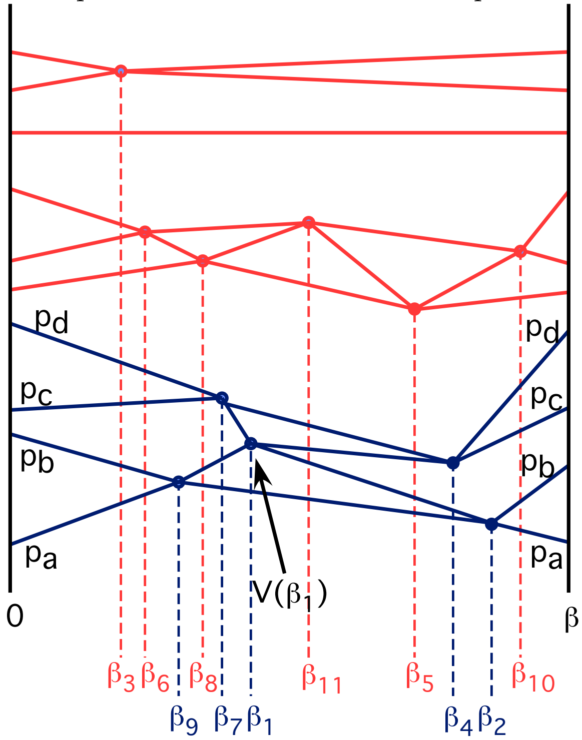
1. The diagram represents a perturbative calculation of the partition function.



   (a) Consider the connected diagram involving $p_a \rightarrow p_d$. When calculating the pressure, this diagram contributes to order $n =$_______ in perturbation theory.

   (b) When performing a virial expansion (see expansion below), the lowest m for which this contributes to A_m is _______.

2. Consider the state:

$$|\eta\rangle = e^{(\eta a^\dagger - \eta^* a)}|0\rangle.$$

Find the overlap $\langle 1|\eta\rangle$.
Hint: you may want to use the Campbell-Baker-Hausdorff lemma – If operators A and B commute to a number,

$$e^{A+B} = e^A e^B e^{-[A,B]/2}$$

3. Consider a virial expansion for a non-relativistic TWO-dimensional gas of spin-1/2 Fermions of mass m at temperature T,

$$\frac{P}{\rho T} = 1 + \sum_{m=2}^{\infty} A_m \left(\frac{\rho}{\rho_0}\right)^{m-1}, \qquad \rho_0 \equiv \frac{1}{(2\pi\hbar)^2} \int_{-\infty}^{\infty} 2\pi p \; dp \; e^{-p^2/2mT}.$$

Ignoring interactions between the particles, calculate A_2. Here ρ is the number per unit length. Begin with the expression for the two-dimensional "pressure", and the density

$$\frac{PA}{T} = \ln Z = \frac{2A}{(2\pi\hbar)^2} \int_{-\infty}^{\infty} 2\pi p \; dp \; \ln\left(1 + e^{-\beta(p^2/2m - \mu)}\right),$$

$$\rho = \frac{1}{A} \frac{\partial}{\partial \beta\mu} \ln Z.$$