Physics 831 Quiz #8 - Monday, Nov. 18, 2013

your name___________________________

1. (10 pts) Consider a virial expansion for a non-relativistic **TWO**-dimensional gas of spin-1/2 fermions of mass m at temperature T.

$$\frac{P}{\rho T} = 1 + \sum_{m=2}^{\infty} A_m \left(\frac{\rho}{\rho_0}\right)^{m-1}, \qquad \rho_0 \equiv \frac{1}{(2\pi\hbar)^2}\int_0^\infty 2\pi p dp \; e^{-p^2/2mT}.$$

Ignoring interactions between the particles, calulate A_2. Here ρ is the number per unit area. Begin with the expressions for the two-dimensional "pressure", and the density

$$\begin{aligned} \frac{PA}{T} &= \ln Z = \frac{2A}{(2\pi\hbar)^2}\int_0^\infty 2\pi p dp \; \ln\left(1 + e^{-\beta(p^2/2m-\mu)}\right), \\ \rho &= \frac{1}{A}\frac{\partial}{\partial \beta\mu}\ln Z. \end{aligned}$$

2. Consider the example for which the surface energy was calculated, where

$$\Delta\Psi \equiv P_0 - P + (\mu - \mu_0)\rho = \frac{A}{2}\left[(\rho - \rho_c)^2 - \alpha^2\right]^2.$$

Using,

$$\begin{aligned} \frac{d}{d(\partial_x \rho)}\left[\frac{P_0 - P + (\mu - \mu_0)\rho}{\partial_x \rho} + \frac{\kappa}{2}\partial_x \rho\right] &= 0, \\ \partial_x \rho &= \sqrt{2\frac{P_0 - P + (\mu - \mu_0)\rho}{\kappa}}, \\ \Delta F/A &= \sqrt{2\kappa}\int_{\rho_{\rm gas}}^{\rho_{\rm liq}} d\rho \; \sqrt{P_0 - P + (\mu - \mu_0)\rho}, \end{aligned}$$

solve for the density profile $\rho(x)$ between the two phases.