your name(s)____

Physics 831 Quiz #4
Friday, Sep. 29, 2017

Work in groups of three to four.
Consider the equation of state

$$P(\rho, T) = \rho T \left(1 + \frac{\rho^2}{\rho_0^2}\right) - a\frac{\rho^2}{\rho_0}.$$

1. (5 pts) Solve for the critical density ρ_c and the critical temperature T_c in terms of ρ_0 and a.
2. (5 pts) Using the Maxwell relation,

$$\left.\frac{\partial(P/T)}{\partial\beta}\right|_{N,V} = -\left.\frac{\partial E}{\partial V}\right|_{N,T},$$

   Find the energy per particle as a function of temperature and density.
3. (5 pts) If the system expands at constant temperature from volume per particle v_a to v_b, find the change in entropy per particle s.
4. (5 pts) Using $Ts = e + Pv - \mu$, find the change in chemical potential between the two points a and b.
5. (5 pts) Find the density of the liquid on the coexistence curve as $T \to 0$.
6. (5 pts) Find the latent heat per particle as $T \to 0$.

Solutions

1.

$$\begin{aligned}
P &= \rho T\left(1+\frac{\rho^2}{\rho_0^2}\right)-a\frac{\rho^2}{\rho_0},\\
\partial P &= 0 = T\left(1+3\frac{\rho^2}{\rho_0^2}\right)-2a\frac{\rho}{\rho_0},\\
\partial^2 P &= 0 = 6T\frac{\rho}{\rho_0^2}-\frac{2a}{\rho_0},\\
T_c &= \frac{a\rho_0}{3\rho_c},\\
\frac{a\rho_0}{3}\left(1+3\frac{\rho_c^2}{\rho_0^2}\right)-2a\frac{\rho_c^2}{\rho_0} &= 0,\\
\rho_c^2\left(\frac{3}{\rho_0}-\frac{2}{\rho_0}\right)+\frac{\rho_0}{3} &= 0,\\
\rho_c &= \frac{\rho_0}{\sqrt{3}},\quad T_c=\frac{a}{\sqrt{3}}.
\end{aligned}$$

2.

$$\begin{aligned}
E &= \frac{3}{2}T-\int_\infty^V dv\ \frac{a\rho^2}{\rho_0}dv\\
&= \frac{3}{2}T-\int_0^\rho \frac{d\rho}{\rho^2}\frac{a\rho^2}{\rho_0}\\
&= \frac{3}{2}T-a\frac{\rho}{\rho_0}.
\end{aligned}$$

3.

$$\begin{aligned}
dS &= \beta dE+(P/T)dV\\
\Delta(S/N) &= -\frac{a}{T}\left(\frac{1}{\rho_0 v_b}-\frac{1}{\rho_0 v_a}\right)+\int dv\left[\rho\left(1+\frac{\rho^2}{\rho_0^2}\right)-\frac{a}{T}\frac{\rho^2}{\rho_0}\right]\\
dv &= -d\rho/\rho^2,\\
\Delta(S/N) &= \ln(v_b/v_a)-\frac{1}{2}\left(\frac{1}{v_b^2\rho_0^2}-\frac{1}{v_a^2\rho_0^2}\right).
\end{aligned}$$

4.

$$\begin{aligned}
\Delta\mu &= \Delta e+\Delta(Pv)-T\Delta(S/N)\\
&= -2a\left(\frac{1}{\rho_0 v_b}-\frac{1}{\rho_0 v_a}\right)+\frac{3T}{2}\left(\frac{1}{\rho_0^2 v_b^2}-\frac{1}{\rho_0^2 v_a^2}\right)-T\ln\left(\frac{v_b}{v_a}\right).
\end{aligned}$$

5. As $T\to 0$, one needs to find liquid in coexistence with a vapor at zero density, thus at zero pressure.

$$P = \rho T\left(1+\frac{\rho^2}{\rho_0^2}\right)-a\frac{\rho^2}{\rho_0^2}, \tag{1}$$

$$\frac{\rho^2}{\rho_0^2}-\frac{a\rho}{T\rho_0}+1 = 0 \tag{2}$$

$$\rho = \rho_0\left[\frac{a}{2T}\pm\sqrt{\frac{a^2}{T^2}-4}\right] \tag{3}$$

This goes to ∞ as $T \to 0$.

6. The energy has a term $-a\rho/\rho_0$, so it also blows up.