Quiz 2 solutions

1. (10 pts) Beginning with the fundamental thermodynamic relation,

$$TdS = dE - \mu dQ + PdV,$$

derive the following Maxwell relation

$$\left.\frac{\partial E}{\partial S}\right|_{\mu,V} = T - \mu \left.\frac{\partial T}{\partial \mu}\right|_{S,V}.$$

$$\text{①} \quad d(E - \mu Q) = TdS - PdV + Q\,d\mu$$

$$T = \left.\frac{\partial (E - \mu Q)}{\partial S}\right|_{\mu, V}$$

$$\left.\frac{\partial E}{\partial S}\right|_{\mu V} - T = -\left.\frac{\partial (\mu Q)}{\partial S}\right|_{\mu V} \stackrel{?}{=} -\mu \left.\frac{\partial T}{\partial \mu}\right|_{S, V}$$

$$\left.\frac{\partial Q}{\partial S}\right|_{\mu V} \stackrel{?}{=} \left.\frac{\partial T}{\partial \mu}\right|_{S V}$$

From Eq. 1, one can see this is true, so

$$\left.\frac{\partial E}{\partial S}\right|_{\mu V} - T = -\mu \left.\frac{\partial T}{\partial \mu}\right|_{S, V} \quad \checkmark$$

2. (10 pts) In Sec. 1.10 of the lecture notes, we showed the force acting on a particle,

$$f = -\left.\frac{\partial E}{\partial x}\right|_{N,S,V},$$

could equivalently be written as

$$f = -\left.\frac{\partial F}{\partial x}\right|_{N,T,V}.$$

where $F = E - TS$ was the Helmholtz free energy. Show that one can extend this to

$$-\left.\frac{\partial F}{\partial x}\right|_{N,T,V} = -\left.\frac{\partial(E - TS - \mu N)}{\partial x}\right|_{\mu,T,V}.$$

You may wish to remember that

$$\mu = \frac{\partial F(N,T,V)}{\partial N}.$$

Just as an aside, $E - TS = -T \ln Z_{\text{can.}}$, and $E - TS - \mu N = -T \ln Z_{\text{G.C.}}$

$$dE = T\,dS - P\,dV + \mu\,dN - f\,dx$$

$$f = \left.\frac{\partial E}{\partial x}\right|_{S,V,N}$$

$$d(E - TS - \mu N) = -S\,dT - N\,d\mu - P\,dV - f\,dx$$

$$f = -\left.\frac{\partial}{\partial x}(E - TS - \mu N)\right|_{T,\mu,V}$$

$$= -\partial_x (PV)\Big|_{T,\mu,V}$$