your name(s)____________________________________

Physics 831 Quiz #5
Friday, Oct. 6, 2017

Work in groups of three to four.
Consider the equation of state

$$P(\rho, T) = \rho T e^{\rho/\rho_0} - a\frac{\rho^2}{\rho_0}.$$

1. (5 pts) Solve for the critical density ρ_c and the critical temperature T_c in terms of ρ_0 and a.

2. (5 pts) Using the Maxwell relation,

$$\left.\frac{\partial(P/T)}{\partial\beta}\right|_{N,V} = -\left.\frac{\partial E}{\partial V}\right|_{N,T},$$

   Find the energy per particle as a function of temperature and density.

3. (5 pts) If the system expands at constant temperature from volume per particle v_a to v_b, find the change in entropy per particle s.

4. (5 pts) Using $Ts = e + Pv - \mu$, find the change in chemical potential between the two points a and b.

5. (5 pts) Find the density of the liquid on the coexistence curve as $T \to 0$.

6. (5 pts) Find the latent heat per particle as $T \to 0$.

1) $P/\rho_0 = xTe^{x} - ax^2, \quad x = \rho/\rho_0$

$\frac{d}{dx}\frac{P}{\rho_0} = Te^{x}\{1+x\} - 2ax = 0$

$\frac{d^2}{dx^2}\frac{P}{\rho_0} = Te^{x}\{2+x\} - 2a = 0$

$2ax = (1+x)\frac{2a}{2+x}$

$x^2 + 2x - 1 = 0, \quad x = \frac{\sqrt{5}-1}{2}, \quad \rho_c = \rho_0\frac{\sqrt{5}-1}{2}$

$T_c = \frac{2a e^{-\rho_c/\rho_0}}{2 + \rho_c/\rho_0}$

$$\frac{\partial(P/T)}{\partial\beta}\bigg|_{N,V} = -\frac{\partial E}{\partial V}\bigg|_{N,T},$$

2.

$$\frac{\partial(E/N)}{\partial(V/N)} = -\partial_\beta\left[-a\beta\frac{\rho^2}{\rho_0}\right] = \frac{a}{\rho_0 v^2}$$

$$E/N = \frac{3}{2}T + \int_\infty^v dv\,\frac{a}{\rho_0 v^2} = \frac{3}{2}T - a\rho/\rho_0$$

3.

$$ds = \beta\, dE/N + \beta P\, dv$$

$$= \beta\left(a\frac{dv}{\rho_0 v^2} + P\,dv\right)$$

$$s_b - s_a = \beta\int_{v_a}^{v_b} dv\left\{\frac{a}{\rho_0 v^2} - \frac{a}{\rho_0 v^2} + (T/v)\,e^{1/\rho_0 v}\right\}$$

$$= -\beta\int d\rho\,\frac{T}{\rho}e^{\rho/\rho_0}$$

$$= \int_{x_a}^{x_b} dx\,\frac{e^x}{x},\quad x_{a,b} \equiv \frac{1}{\rho_0 v_{a,b}}$$

$$\int_{x_a}^{x_b}\frac{e^x}{x}dx = \int_{-x_a}^{-x_b}\frac{e^{-x}}{x}dx = E_i(x_a) - E_i(x_b)$$

$$s_b - s_a = E_i(x_b) - E_i(x_a)$$

4.

$$Ts = (E/N) + P/\rho - \mu$$

$$\Delta\mu = \Delta E/N + \Delta P/\rho - \Delta(Ts)$$

$$= \frac{2a\rho_a}{\rho_0} - \frac{2a\rho_b}{\rho_0} + T\left(\rho_b e^{\rho_b/\rho_0} - \rho_a e^{\rho_a/\rho_0}\right)$$

$$- TE_i(\rho_b/\rho_0) + TE_i(\rho_a/\rho_0)$$

5. You need to find ρ_L s.t.

$$\rho_L T e^{\rho_L/\rho_0} \to a\rho_L^2 = \text{finite as } T \to 0,$$

only if $\rho_L \to \infty$.

6. $L = \infty$ because $E/N \sim -a\rho^2/\rho_0$