your name(s)____

Physics 831 Quiz #6 - Friday, Oct. 13 2017

Work in groups of three or four:

1. Consider an engine that works in a PV cycle $a \to b \to c \to d \to a \cdots$. The equation of state is that of a monotonic ideal gas.

$$
\begin{aligned}
P_a = P_b = 2P_0, &\qquad P_c = P_d = P_0 \\
V_a = V_d = V_0, &\qquad V_b = V_c = 2V_0.
\end{aligned}
$$

   (a) (5 pts) What is the work done in the cycle?

   (b) (5 pts) What is the efficiency of the engine?

2. An ideal gas of particles of mass m is initially: at a uniform temperature T_0, has zero collective velocity, and as far as the eye can see, the number density profile initially has an exponential profile in the x direction:

$$\rho(x, t = 0) = \rho_0 e^{-x/\lambda}.$$

   The gas then expands expands hyrodynamically. For a solution, assume the velocity is independent of x and rises linearly in time, $v = \alpha t$.

   (a) (5 pts) Find α.

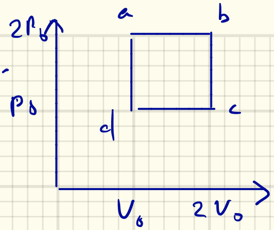
   (b) (5 pts) What is the temperature, $T(x, t)$?

   (c) (5 pts) What is the density profile, $\rho(x, t)$?

   The equations of hydrodynamics are:

$$
\begin{aligned}
\frac{D}{Dt} v &= -\frac{\partial_x P}{m\rho}, \\
\frac{D}{Dt} \rho &= -\rho \partial_x v, \\
\frac{D}{Dt} \epsilon &= -(P + \epsilon) \nabla \cdot \mathbf{v}, \\
\frac{D}{Dt} &= \partial_t + v \partial_x.
\end{aligned}
$$

1.



a) W = area in loop

$= P_0 V_0$

b) Heat enters in ab & da

$$\text{eff} = \frac{P_0 V_0}{Q_{da} + Q_{ab}}$$

$$U_a = \frac{3}{2} \cdot 2 P_0 V_0 , \quad U_b = \frac{3}{2} \cdot 4 P_0 V_0 , \quad U_d = \frac{3}{2} P_0 V_0$$

$$Q_{ab} = U_b - U_a + W_{ab} = 3 P_0 V_0 + 2 P_0 V_0 = 5 P_0 V_0$$

$$Q_{da} = U_a - U_d = \frac{3}{2} P_0 V_0$$

$$e = \frac{13/2 \; P_0 V_0}{P_0 V_0} = \frac{13}{2}$$

2a) $\alpha = \dfrac{-\partial_x \rho T}{m\rho} = T_0/m\lambda$

b) $T = T_0$, entropy conserved, no expansion

c) $\rho = \rho_0 \exp\left\{-\left(x - \frac{1}{2}\alpha t^2\right)/\lambda\right\}$