A problem concerning free energy

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1. Starting from the fundamental thermodynamic relation $dE = TdS - pdV + \mu dN$, express the following quantities in terms of the derivatives of Helmholtz Free Energy $F = F(T, V, N)$.

   - Entropy S, pressure P and the chemical potential μ
   - Energy E and Gibbs' Free Energy $G \equiv E - TS + pV$
   - The Grand Potential $\Omega \equiv E - TS - \mu N$.

2. Recall that the partition functions for canonical and grand canonical ensembles can be constructed from Helmholtz Free Energy and the Grand Potential, respectively,

$$F = -T \ln Z_{\mathrm{C}},\ \Omega = -T \ln Z_{\mathrm{G.C.}} \tag{1}$$

   From the results of 1, derive a relationship between $\ln Z_{\mathrm{C}}$ and $\ln Z_{\mathrm{G.C.}}$.

3. Compare and explain the difference between the result in 2 and the standard result in statistical mechanics (2).

$$Z_{\mathrm{G.C.}} = \sum_{N} \mathrm{e}^{-\alpha N} Z_{\mathrm{C}}(N) \tag{2}$$

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