Important Equations:
Generalized Equipartition Theorem: $\langle q\frac{\partial H}{\partial q}\rangle = T$
Virial Theorem: $\langle q_i\frac{\partial H}{\partial q_i}\rangle = \langle p_i\frac{\partial H}{\partial p_i}\rangle$

Name:____

1. Using the virial theorem and the non-generalized equipartion theorem, show that for a nonrelativistic particle moving in a three-dimensional potential,

$$V(r) = Kr^N \tag{1}$$

The average potential energy is

$$\langle V(r)\rangle = \frac{3T}{N} \tag{2}$$

1 Solution:

First we rewrite the potential in Cartesian coordinates

$$V(r) = K(\sqrt{x^2+y^2+z^2})^N = K(x^2+y^2+z^2)^{\frac{N}{2}} \tag{3}$$

We then use the Generalized equipartition theorem, recognizing that we get a factor of T for each dimension

$$\langle q_i\frac{\partial V(r)}{\partial q_i}\rangle = 3T \qquad \langle q_i\frac{\partial V(r)}{\partial q_i}\rangle = \langle KNq_i^2(x^2+y^2+z^2)^{(\frac{N}{2}-1)}\rangle \tag{4}$$

Where q_i is x, y, and z.
We then sum over i to get

$$\sum_i\langle q_i\frac{\partial V(r)}{\partial q_i}\rangle = \langle KN(x^2+y^2+z^2)(x^2+y^2+z^2)^{(\frac{N}{2}-1)}\rangle = \langle KN(x^2+y^2+z^2)^{(\frac{N}{2})}\rangle \tag{5}$$

We recognize this as N times the original potential, and divide both sides of the Generalized Equipartition Theorem by N to get our final answer

$$\langle KN(x^2+y^2+z^2)^{(\frac{N}{2})}\rangle = N\langle Kr^N\rangle = 3T \qquad \langle V(r)\rangle = \frac{3T}{N} \tag{6}$$