Solutions to Section (2.4 - 2.5) Questions

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1 Solution to Problem 1

$$\rho = \frac{(2s+1)}{(2\pi\hbar)^D}\int d^D p f \tag{1}$$

$$f = \frac{e^{-\beta(\epsilon-\mu)}}{1-e^{-\beta(\epsilon-\mu)}} \tag{2}$$

$s = 0$, $pc = \epsilon$, $D = 3$ a bose condensate is formed when $\mu \Rightarrow 0^-$

$$\rho = \frac{1}{(2\pi\hbar)^3}\int d^3 p \frac{e^{-\beta(pc-\mu)}}{1-e^{-\beta(pc-\mu)}} \Rightarrow \frac{4\pi}{(2\pi\hbar)^3}\int_0^\infty dp p^2 \frac{e^{-\beta pc}}{1-e^{-\beta pc}} \tag{3}$$

When we evaluate this integral we get:

$$\rho = \frac{4\pi}{(2\pi\hbar)^3}\frac{2\zeta(3)}{\beta^3 c^3} \tag{4}$$

This can be condensed to:

$$\rho = (\pi^{2/3}\hbar\beta c)^{-3}\zeta(3) \tag{5}$$

2 Solution to Problem 2

$$\frac{PV}{T} = V\frac{(2s+1)}{(2\pi\hbar)^3}\int d^3 p ln(1+e^{-\beta(\epsilon-\mu)}+\cdots) \tag{6}$$

We can also see that:

$$d^3 p = 4\pi p^2 dp = \frac{d}{dp}(\frac{p^3}{3}) \tag{7}$$

$$P = 4\pi T\frac{(2s+1)}{(2\pi\hbar)^3}\int_0^\infty \frac{p^3}{3}dp\frac{d}{dp}ln(1+e^{-\beta(\epsilon-\mu)}+\cdots) \tag{8}$$

We know for nonreletivistic particles $\epsilon = \frac{p^2}{2m}$, $s = \frac{1}{2}$

$$4\pi T\frac{(2)}{(2\pi\hbar)^3}\int_0^\infty dp\frac{p^4}{3\epsilon T}f(\epsilon) \tag{9}$$

$$P = \frac{2}{3}\frac{m}{\pi^2\hbar^3}\int_0^\infty dp\frac{e^{-\beta(p^2/2m-\mu)}}{1-e^{-\beta(p^2/2m-\mu)}} \tag{10}$$