

$$a) \rho_0 = \frac{(2s+1)}{(2\pi\hbar)^2} \left[\int d^2p e^{-\frac{p^2}{2mT}} \right]^2 = \frac{mT}{\hbar^2 \pi}$$

$$b) \rho = \frac{(2s+1)}{(2\pi\hbar)^2} \int d^2p \left\{ e^{-\frac{p^2}{2mT} + \beta\mu} - e^{-\frac{2p^2}{2mT} + 2\beta\mu} \right\}$$

$$= \rho_0 \left(e^{\beta\mu} - \frac{1}{2} e^{2\beta\mu} \right) \quad \text{where we used } \frac{x}{1+x} \approx x - x^2 \text{ on } f(p)$$

c) To second order,

$$\rho^2 \approx \rho_0^2 e^{2\beta\mu} \quad (\text{cross terms are higher order})$$

$$d) \rho = \frac{T(2s+1)}{(2\pi\hbar)^2} \int d^2p \ln \left\{ 1 + e^{-\beta \left(\frac{p^2}{2m} - \mu \right)} \right\}$$

$$\approx \frac{T(2)}{(2\pi\hbar)^2} \int d^2p \left(e^{-\beta \left(\frac{p^2}{2m} - \mu \right)} - \frac{1}{2} e^{-2\beta \left(\frac{p^2}{2m} - \mu \right)} \right)$$

↑ since $\ln(1+x) \approx x - \frac{1}{2}x^2 + \dots$

$$= T \rho_0 e^{\beta\mu} - \frac{1}{4} T \frac{\rho_0^2}{\rho_0} e^{2\beta\mu}$$

$$\rightarrow \delta\rho = \frac{1}{4} T e^{2\beta\mu} \frac{\rho_0^2}{\rho_0} = \frac{1}{4} T \frac{\rho_0^2}{\rho_0}$$

e) From $\delta\rho$, it can be ~~seen~~ seen that A_2 is $\frac{1}{4}$