Section 3.5

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Calculate the second order virial coefficient for a gas of distinguishable non-relativistic particles of mass m at temperature T that interact through a hard core potential,

$$V(x) = \begin{cases} \infty, & r < a \\ 0, & r > a \end{cases} \tag{1}$$

Consider any l - wave contribution (valid at low T).

1 Solution

$$\frac{qR}{\hbar} - l\pi + \delta_l = n\pi, n = 1, 2, 3, ...$$

$$\delta_l = (n + l)\pi - \frac{qR}{\hbar}$$

$$\frac{\partial \delta_l}{\partial \epsilon} = \frac{\partial \delta_l}{\partial q}\frac{\partial q}{\partial \epsilon} = \frac{-R}{\hbar}\frac{\partial q}{\partial \epsilon}$$

We subsitute this into our formula for the Virial Coefficient

$$A_2 = -2^{3/2}\sum_l \int d\epsilon \frac{2l+1}{\pi}\frac{d\delta_l}{d\epsilon}e^{-\epsilon/T}$$

$$E = \frac{q^2}{2m}$$

$$A_2 = \frac{2^{3/2}R}{\pi\hbar}\int_{-\infty}^{\infty} dq e^{-q^2/2mT}\sum_l (2l+1)$$

$$= \frac{2^{3/2}R}{\pi\hbar}\sqrt{2mT\pi} = \frac{4R}{\hbar}\sqrt{\frac{mT}{\pi}}(2l+1)$$