PHY 831 : Statistical Mechanics

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<u>Chapter 6.1-6.4</u>

Problem

Assume that the free energy in a two-dimensional system obeys the following form,

$$F = \int d^2r \left(\frac{A}{2}\phi^2 + \frac{C}{2}\phi^6 \right)$$

Assuming that near T_c, $A \sim at$, find the critical exponent in mean field theory β where,

$$\langle \phi \rangle \sim t^\beta$$

below T_c.

Solution

Free energy (F) is the integration of free energy density (f) over all area. Hence,

$$f = \left(\frac{A}{2}\phi^2 + \frac{C}{2}\phi^6 \right)$$

Now, free energy density is minimized for the mean field $\langle \phi \rangle$. So taking a derivative, we get

$$\frac{df}{d\phi} = 0$$

$$\implies \frac{d}{d\phi} \left(\frac{A}{2}\phi^2 + \frac{C}{2}\phi^6 \right) = 0$$

$$\implies A\phi + 3C\phi^5 = 0$$

The solutions are $\phi = 0$, $\phi = \pm \left(\frac{-A}{3C} \right)^{\frac{1}{4}}$. Since, $\phi = 0$ is a maxima, we discard it. Now, using $A \sim at$ for T_c, we get,

$$\langle \phi \rangle = \pm \left(\frac{-at}{3C} \right)^{\frac{1}{4}}$$

So, it is clear that,

$$\langle \phi \rangle \sim t^{\frac{1}{4}}$$

Hence, $\beta = \frac{1}{4}$ is obtained.

The above can be performed for any form of f for any number of dimensions.