your name(s)__________________________

Physics 841 Quiz #2 - Monday, Jan. 30

Work in groups of four or fewer. This is open-note, open-book, open-mouth, open-internet, and open-mind.

Turn in one worksheet per group, with all names included.

1. Consider a region with a magnetic field, $A_y = Bx$, which gives a magnetic field in the $\hat{z}$ direction.

   (a) Consider a boost in the $\hat{y}$ direction by velocity v. Find the new electric and magnetic fields $\vec{E}'$ and $\vec{B}'$.

   (b) What is $|\vec{B}'|^2 - |\vec{E}'|^2$?

   (c) Are there any reference frames in which the magnetic field vanishes?

Solution:
a)

$$B'_z = \gamma B, \tag{1}$$
$$E'_x = \gamma v B. \tag{2}$$

b) B^2
c)No

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2. Consider a region with both a magnetic field $\vec{B} = B\hat{z}$ and an electric field $\vec{E} = E\hat{x}$, and $|B| > |E|$.

   (a) Write out the electromagnetic field tensor, $F^{\alpha\beta}$.

   (b) Beginning with the equations
   $$m\frac{d}{d\tau}u^\alpha = qF^{\alpha\beta}u_\beta,$$
   write the equations of motion for u_x, u_y and u_z, in terms of $d/d\tau$, where τ is the time measured in the frame of the particle. The equations should involve E and B, rather then $F^{\alpha\beta}$. Assume the particle has charge q and mass m.

   (c) Find solutions for $x'(t')$, $y'(t')$ and $z'(t')$, where the primes denote that you are in the frame where there is no electric field. Assume the initial conditions were set up so that $u'_z = 0$ and motion is circular with the center of the circle at the origin, with $x'(t' = 0) = R$, and assume the charge q is positive. Express your answer in terms of R, $B' = \sqrt{B^2 - E^2}$, m, q and τ. Be sure to show how the frequency of the motion depends on R, B' and q.

   (d) Going back to the original frame, where there is also an electric field, find $x(t')$, $y(t')$ and also $t(t')$. (Note it would be difficult to express $x(t)$ and $y(t)$ in closed form.)

   (e) For very large times find $\bar{x}(t)$ and $\bar{y}(t)$ averaged over an oscillation period. I.e. only find the dependence that grows with time. With what velocity does the point $(\bar{x}(t), \bar{y}(t))$ move? How is this answer related to the velocity required to boost away the electric field?

Solution:

a)

$$F^{\alpha\beta} = \begin{pmatrix} 0 & E & 0 & 0 \\ -E_x & 0 & -B & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

b)

$$\begin{aligned} \partial_\tau u_x &= qEu_0 + qBu_x, \\ \partial_\tau u_y &= -qBu_x \end{aligned}$$

c)

$$\begin{aligned} x' &= R\cos(\omega' t' + \phi'), \quad \omega' = \frac{qB}{m\gamma_\omega}, \quad \gamma_\omega = 1/\sqrt{1-\omega'^2 R^2}. \\ y' &= -R\sin(\omega' t' + \phi') \\ t' &= \gamma(t - vy), \quad v = E/B, \gamma = 1/\sqrt{1-v^2}. \end{aligned}$$

d) boost in $\hat{y}$ direction,

$$\begin{aligned} x &= x' \\ y &= \gamma y' - \gamma v t', \\ t &= \gamma t' - \gamma v y'. \end{aligned}$$

e)

$$\begin{aligned} y &= \gamma y' + v(\gamma t - \gamma v y), \\ y(1+\gamma^2 v^2) &= \gamma y' + \gamma^2 v t, \\ y &= \frac{y'}{\gamma} + vt. \end{aligned}$$

Because y' oscillates, $\bar{y} = vt$, with $v = E/B$.