# *FINAL EXAM*

## PHYSICS 851, FALL 1999

1. A magnetic field is applied 60° to the $z$ axis with the time dependence,

$$B(t) = \begin{cases} 0, & t < 0 \\ B_0, & 0 < t < \tau \\ 0, & t > \tau \end{cases}$$

   The magnetic field interacts with a spin-1/2 particle through the interaction

$$H = \alpha \mathbf{S} \cdot \mathbf{B}$$

   If the particle is initially in a spin-up state (up being defined relative to the $z$ axis), find the probability of the particle being in the spin-down state for times, $t > \tau$.

2. A spin 1/2 particle has positive spin along the $y$ axis.

   (a) Write down the density matrix $\rho$ for this state. Use the basis where $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ denotes a state with positive spin along the $z$ axis, and $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$ refers to a particle with a positive spin projection along the $y$ axis.

   (b) Find $\rho^2$.

   (c) Write down the density matrix for the state that is an incoherent mixture of 50% positive spin along the $y$ axis and 50% negative spin along the $y$ axis.

   (d) What is the square of this density matrix?

3. Consider a particle of mass $m$ that feels an attractive one-dimensional delta function potential,

$$V(x) = \beta\delta(x)$$

   A particle of mass $m$ approaches the origin with energy $E$. What fraction of the incident flux is reflected?

4. Express the state $|j_1 = 1/2, j_2 = 1, m_1 = 1/2, m_2 = 0\rangle$ as a linear combination of eigenstates of total angular momentum, $J$ and projection, $M$.

5. A resonant state $R$ with energy $E_R$ can decay into the TWO-DIMENSIONAL continuum states, which are eigenstates of momentum $\hbar\mathbf{k}$, due to an interaction $V$. The matrix element between the resonant state and the continuum state is:

$$\langle R|V|k\rangle = \frac{\alpha(k)}{\sqrt{A}},$$

where $A$ is the large area confining the state $k$. The energy of the continuum state is $\hbar^2 k^2/(2\mu)$, and note that the matrix element above is independent of the direction of $\mathbf{k}$.

Using Fermi's golden rule, estimate the exponential lifetime of the resonance in terms of $\alpha(k)$, $\mu$, $A$ and $E_R$.

6. Consider a harmonic oscillator characterized by the frequency $\omega$, where a perturbation is added,

$$V = \beta\left(a^\dagger a^\dagger + aa\right).$$

Find the correction to the ground state energy in second order perturbation theory.