1. Consider two oscillator levels described by the creation operators, a_1^\dagger and a_2^\dagger , where the Hamiltonian is

$$H=\epsilon_1 a_1^\dagger a_1 + \epsilon_2 a_2^\dagger a_2 + eta (a_1^\dagger a_2^\dagger + a_1 a_2).$$

Consider the operators

$$b_1^\dagger \equiv \cosh \eta \; a_1^\dagger + \sinh \eta \; a_2, \ b_2^\dagger \equiv \cosh \eta \; a_2^\dagger + \sinh \eta \; a_1.$$

- (a) Show that b_i and b_i^{\dagger} behave like creation/destruction operators.
- (b) Find the values of η , E_0 , E_1 and E_2 such that allow H to be written as

$$H=E_0+E_1b_1^{\dagger}b_1+E_2b_2^{\dagger}b_2.$$

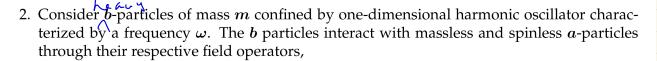
This is known as a Bogoliubov transformation.

a)
$$\begin{bmatrix} b, b \\ + \end{bmatrix} = \begin{bmatrix} \omega s h m a_1 + s i n h m a_2 \\ \omega s h m - s i n h m = 1 \end{bmatrix}$$

$$= \omega s h m - s i n h m a_1 + s i n h m a_2 \\ b + \end{bmatrix} = \begin{bmatrix} \omega s h m a_1 + s i n h m a_2 \\ \omega s h m a_2 + s i n h m a_3 \end{bmatrix}$$

$$= \omega s h m s | n h m a_2 + s i n h m a_4 \\ b + h + s i n h m a_4 + s i n h m a_4 \end{bmatrix}$$

$$= \omega s h m - s i n h m a_2 + s i n h m a_2 \\ \omega s h m a_2 + s i n h m a_4 \\ \omega s h m a_4 + s i n h m a_4 \\ \omega s h m a_4 + s i n h m a_4 \\ \omega s h m a_4 + s i n h m a_4 \\ \omega s h m a_4 + s i n h m a_4 \\ \omega s h m a_4 + s i n h m a_4 \\ \omega s h m a_4 + s i n h m a_4 \\ \omega s h m a_4 + s i n h m a_5 \\ \omega s h m a_4 + s i n h m a_4 \\ \omega s h m a_4 + s i n h m a_5 \\ \omega s h m a_4 + s i n h m a_5 \\ \omega s h m a_4 + s i n h m a_5 \\ \omega s h m a_4 + s i n h m a_5 \\ \omega s h m a_4 + s i n h m a_5 \\ \omega s h m a_5$$

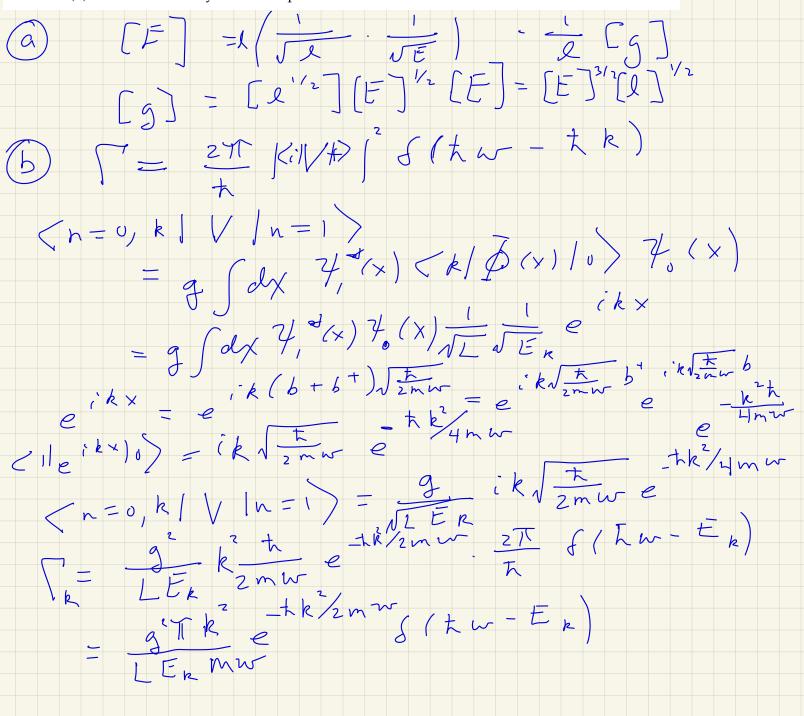


$$H_{
m int} = g \, \int dx \Psi^\dagger(x) \Phi(x) \Psi(x),$$

where Φ and Ψ are the field operators for the a-particles and b-particles respectively.

$$egin{align} \Phi(x) &= rac{1}{\sqrt{L}} \sum_k rac{1}{\sqrt{2\!\!/\!2\!E_k}} \left(e^{ikx} a_k^\dagger + e^{-ikx} a_k
ight) \ \Psi^\dagger(x) &= rac{1}{\sqrt{L}} \sum_k e^{ikx} b_k^\dagger \ \end{aligned}$$

- (a) What are the dimensions of g?
- (b) What is the decay rate of a *b* particle in the first excited state.



= gk _ e the /z mw = mErtcw $= 3^{2} + \frac{1}{2} k^{2} / 2 m w$ $= m t^{2} c^{3} e$

3. Show that Eq. (9.33) is satisfied by using the electric and magnetic fields defined in Eq. (9.31).

$$ec{A}(ec{r},t) = \sqrt{rac{2\pi\hbar^2c^2}{V}} \sum_{k,s} ec{\epsilon}_s(ec{k}) rac{1}{\sqrt{E_k}} \left(e^{iec{k}\cdotec{r}-iE_kt/\hbar} a_{k,s} + e^{-iec{k}\cdotec{r}+iE_kt/\hbar} a_{k,s}^\dagger
ight)$$

Let
$$\frac{1}{2}$$
 $\frac{3\pi}{4}$ = $\frac{1}{2}$ $\frac{3\pi}{4}$ = $\frac{1}{2}$ = $\frac{1}{2$

4. A proton in a nucleus decays from an excited state to its ground state by emitting a photon of momentum $\hbar \vec{k}$ and polarization ϵ_s . The matrix element describing the decay is

$$\langle 0,k,s|V|1
angle = eta ec{\epsilon}_s \cdot \int d^3r rac{e^{iec{k}\cdotec{r}}}{\sqrt{V}} \left(\phi_0^*(ec{r})
abla \phi_1(ec{r}) - [
abla \phi_0^*(ec{r})]\phi_1(ec{r})
ight).$$

The factor β absorbed all the various factors involved in defining the vector field in Eq. (9.31). Assume the ground and excited states are modeled with a three-dimensional harmonic oscillator of frequency ω . If the excited state is in the first level of a harmonic oscillator and has an angular momentum projection m, what is the angular distribution, $d\Gamma/d\Omega$

of the decay for each m. Remember that the two polarizations of the photon must be perpendicular to \vec{k} . You need only calculate the angular shape of the distribution – ignore the prefactors.

$$| h = \sqrt{\frac{1}{2}} | (a - a + \frac{1}{2}) | (a -$$

5. A spinless particle of mass m and charge e is in the first excited state of a three-dimensional harmonic oscillator characterized by a frequency ω . Assume the harmonic oscillator in the Cartesian state with $n_z = 1$. Using the interaction

$$H_{
m int} = ec{j} \cdot ec{A}/c,$$

- (a) Calculate the decay rate of the charged particle into the ground state of the oscillator in the dipole approximation.
- (b) Calculate $d\Gamma/d\Omega$ as a function of the emission angles of the photon, θ and ϕ .
- (c) In terms of the unit vectors \hat{k} , $\hat{\theta}$ and $\hat{\phi}$, write the two polarization vectors which are allowed for emission of a photon at an angle θ , ϕ .
- (d) For each polarization vector above, calculate $d\Gamma_s/d\Omega$, the probability of decaying via emission of a photon emited in the θ , ϕ direction with polarization s.

b)
$$\int d\Omega = \frac{e^{i} v^{i}}{d\Omega} \int d\Omega = \frac{e^{i} v^{i}}{4\pi mc^{3}} \int d\Omega = \frac{e^{i} v^{i}}{3\pi mc^{3}}$$
 $= \frac{e^{i} v^{i}}{mc^{3}} \left(\frac{1-i}{5} \right) = \frac{2}{3} \frac{e^{i} v^{i}}{mc^{3}}$
 $= \frac{e^{i} v^{i}}{4\pi mc^{3}} \left(\frac{1}{2} + \frac{1}{2}$

6. Again consider a spinless particle of mass m and charge e n the first excited state of a three-dimensional harmonic oscillator characterized by a frequency ω . However, this time assume the charged particle is originally in a state with angular momentum projection m=+1 along the z axis. Using the interaction

$$H_{
m int} = ec{j} \cdot ec{A}/c,$$

and applying the dipole approximation,

- (a) Find the decay rate Γ of the first excited state.
- (b) Find the differential decay rate $d\Gamma/d\Omega$.

a) Same as (a) from previous

b)
$$\langle 0| P_{x} - iP_{y} | n=1, m=1 \rangle$$
 $| i m w \langle 0| \frac{x-iy}{\sqrt{z}} | n=1, m=1 \rangle$
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