Isospin?

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Isospin

Weird stuff that mimics spin

Approximate symmetry

- broken for up and down quarks with different masses and charges
- kept for strong interaction H = $\mathbf{H_s}$ + H_{other}

Adds like angular momentum

Use raising and lowering operators

$$I_{\pm}|I, I_z\rangle = \sqrt{I(I+1) - I_z(I_z \pm 1)}|I, I_z \pm 1\rangle$$

Question

a) Write the total isospin state $I=3/2$, $I_z=-\frac{1}{2}$ as a linear combination of pion and nucleon pairs

b) Find the ratio of cross sections for the reactions $\pi^- + p \rightarrow \pi^- + p$

and $\pi^- + p \rightarrow \pi^0 + n$, using that the Hamiltonian must conserve isospin since it is a strong interaction

a) Write the total isospin state I=3/2, I_z=-½ as a linear combination of pion and nucleon pairs

$$I_- = I_{-\pi} + I_{-p}$$

$$|I = 3/2, I_z = 3/2\rangle = |\pi^+, p\rangle$$

$$I_-|3/2, 3/2\rangle = I_-|\pi^+, p\rangle = (I_{-\pi} + I_{-p})|\pi^+, p\rangle$$
$$|3/2, 1/2\rangle = \sqrt{2/3}|\pi^0, p\rangle + \sqrt{1/3}|\pi^+, n\rangle$$
$$I_-|3/2, 1/2\rangle = I_-(\sqrt{2/3}|\pi^0, p\rangle + \sqrt{1/3}|\pi^+, n\rangle)$$
$$|3/2, -1/2\rangle = \sqrt{4/12}|\pi^-, p\rangle + \sqrt{2/12}|\pi^0, n\rangle + \sqrt{2/12}|\pi^0, n\rangle + 0$$
$$|3/2, -1/2\rangle = \sqrt{1/3}|\pi^-, p\rangle + \sqrt{2/3}|\pi^0, n\rangle$$

b) Calculate the ratio of cross section for $\pi^- + p \to \pi^- + p$ vs $\pi^- + p \to \pi^0 + n$ using that the Hamiltonian must conserve isospin since it is a strong interaction

Using Fermi's Golden to look at matrix element and approximating $\sigma \propto |\langle f|S|i\rangle|^2$
Where S is the evolution matrix.

We have that the scattering happens through a Δ^0 baryon, which has I = 3/2
Using that $\langle \pi^-, p|H|\Delta^0\rangle = \frac{1}{\sqrt{2}}\langle \Delta^0|H|\pi^0, n\rangle$ (as we did in problem 10.1)

$$|\langle \pi^-, p\,|S|\,\pi^-, p\rangle|^2 \propto |\langle \pi^-, p|H|\Delta^0\rangle\langle\Delta^0|H|\pi^-, p\rangle|^2 = |\frac{1}{\sqrt{2}}\langle\Delta^0|H|\pi^0, n\rangle\langle\Delta^0|H|\pi^-, p\rangle|^2 = \frac{1\cdot 2\cdot 1}{2\cdot 3\cdot 3} = \frac{1}{9}$$

$$|\langle \pi^-, p\,|S|\,\pi^0, n\rangle|^2 \propto |\langle \pi^-, p|H|\Delta^0\rangle\langle\Delta^0|H|\pi^0, n\rangle|^2 = \left|\frac{1}{\sqrt{2}}\langle\Delta^0|H|\pi^0, n\rangle\langle\Delta^0|H|\pi^0, n\rangle\right|^2 = \frac{1\cdot 2\cdot 2}{2\cdot 3\cdot 3} = \frac{2}{9}$$

The ratio of scattering cross sections are then

$$\frac{\sigma_{\pi^- + p\to\pi^- + p}}{\sigma_{\pi^- + p\to\pi^0 + n}} = \frac{1/9}{2/9} = \frac{1}{2}$$