Quantum Chapter 14: Coherent States

Spring 2020

1 Intro

A **coherent state** (a.k.a. Glauber state) is a state for bosons in a quantum harmonic oscillator. It is an eigenstate of the destruction operator, not the particle number. It is defined as:

$$\begin{aligned} |\eta\rangle &= e^{-\eta^*\eta/2} e^{\eta a^\dagger} |0\rangle \\ &= e^{-\eta^*\eta/2} \sum_n \frac{(\eta a^\dagger)^n}{n!} |0\rangle \end{aligned}$$

Where η is a complex number and $a|\eta\rangle = \eta|\eta\rangle$

Since a coherent state is unchanged by the destruction operator, one can observe particles in the system without changing it. The number distributions of coherent states follow a Poisson distribution, as we'll show in this problem:

2 Problem 14.4

(a) Show that $\bar{N} = \langle\eta| N_{op} |\eta\rangle = \eta^*\eta$, where $N_{op} = a^\dagger a$ is the number operator.

$$\begin{aligned} \langle\eta| N_{op} |\eta\rangle &= \langle\eta| a^\dagger a |\eta\rangle \\ &= (a|\eta\rangle)^\dagger (a|\eta\rangle) \\ &= \eta^*\eta \langle\eta|\eta\rangle \\ &= \eta^*\eta \end{aligned}$$

(b) Show that the variance equals the mean

$$\begin{aligned} \langle\eta| (N_{op} - \bar{N})^2 |\eta\rangle &= \langle\eta| N_{op}^2 |\eta\rangle + \langle\eta| \bar{N}^2 |\eta\rangle - \langle\eta| 2\bar{N} N_{op} |\eta\rangle \\ &= \langle\eta| (a^\dagger a)(a^\dagger a) |\eta\rangle + (\eta^*\eta)^2 - 2\eta^*\eta \langle\eta| N_{op} |\eta\rangle \\ &= \langle\eta| a^\dagger(1 + a^\dagger a) a |\eta\rangle - (\eta^*\eta)^2 \\ &= \eta^*\eta(\langle\eta|\eta\rangle + \langle\eta| a^\dagger a |\eta\rangle) - (\eta^*\eta)^2 \\ &= \eta^*\eta + (\eta^*\eta)^2 - (\eta^*\eta)^2 \\ &= \eta^*\eta \end{aligned}$$