# Quantum Hall Effect

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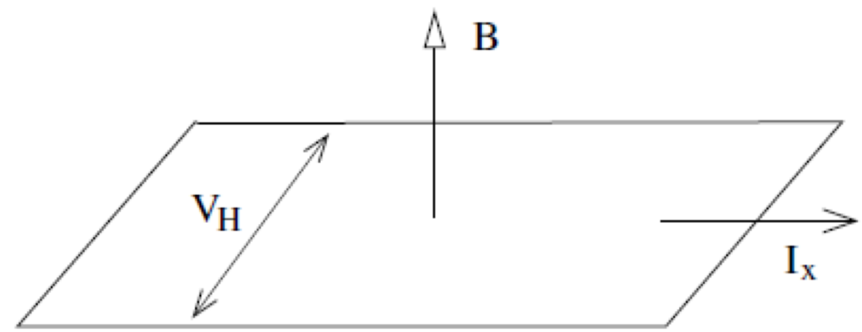
# Classical Hall (for Context)

- Consider a 2-D conducting plane
- Pass a current and apply a magnetic field as in figure
- Electron accumulation after deflection
- Consider classical electron behavior
- Obtain differential equation
- Assert solution as fact
- Find a classical harmonic oscillator!




$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{v} \times \mathbf{B}$$

$$x(t) = X - R\sin(\omega_B t + \phi) \quad \text{and} \quad y(t) = Y + R\cos(\omega_B t + \phi)$$

# The Drude Model of Conductivity

- Add interaction with E field and dissipation to diff-eq
- Look for equilibrium states
- Consider current density instead of v
- Rewrite in terms of matrix
- Obtain conductivity tensor!

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E} - e\mathbf{v} \times \mathbf{B} - \frac{m\mathbf{v}}{\tau}$$

$$\mathbf{J} = -ne\mathbf{v}$$

$$\begin{pmatrix} 1 & \omega_B\tau \\ -\omega_B\tau & 1 \end{pmatrix} \mathbf{J} = \frac{e^2 n\tau}{m}\mathbf{E}$$

$$\mathbf{J} = \sigma\mathbf{E}$$

$$\sigma = \frac{\sigma_{DC}}{1+\omega_B^2\tau^2}\begin{pmatrix} 1 & -\omega_B\tau \\ \omega_B\tau & 1 \end{pmatrix} \quad \text{with} \quad \sigma_{DC} = \frac{ne^2\tau}{m}$$

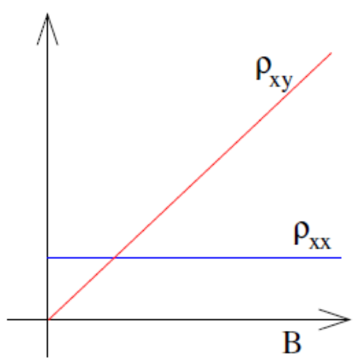
# Resistance vs. Resistivity

- Hall conductivity for context
- Resistivity tensor (inverse of conductivity as always)
- Note: $\rho_{xx}$ differs from resistance by geometrical factors, $\rho_{xy}$ always corresponds with the resistance
- Specific values and finally classical resistivity behavior

$$\sigma_{xy} = \frac{e^2}{2\pi\hbar}\nu$$

$$\rho = \frac{1}{\sigma_{DC}}\begin{pmatrix} 1 & \omega_B\tau \\ -\omega_B\tau & 1 \end{pmatrix}$$

$$\rho_{xx} = \frac{m}{ne^2\tau} \quad \text{and} \quad \rho_{xy} = \frac{B}{ne}$$



# Quantum Hall Effect in the Landau Gauge

- Looking for solutions:
- Consider the Hamiltonian for a particle under the effect of a magnetic field with the Landau gauge
- Hamiltonian can quickly be re-written to resemble a Harmonic oscillator
- Can derive exactly the same result using raising and lowering operators
- (Center offset in the x direction, but normal in y due to choice of Gauge)
- Harmonic oscillator energy levels with no dependence on $p_y$

$$H = \left(\vec{p} - e\vec{A}/c\right)^2/(2m)$$

$$\vec{A} = Bx\hat{y}$$

$$H = \frac{P_x^2}{2m} + \frac{(p_y - eBx/c)^2}{2m}$$

$$H = \frac{P_x^2}{2m} + \frac{1}{2}m\omega^2\left(x - \frac{p_y c}{eB}\right)^2$$

# Filled Landau Levels

- Looking for number of electrons to fill energy levels:
- No dependence on $p_y$ leads to massive degeneracies
- Density of these types of states is given
- But limits on $p_y$ are given by $L_x$ given that $p_y$ is the center of the H.O. in x
- Can obtain values for B for which the plateaus shift in terms of n, m, and universal constants.
- Center of plateau occurs at values proportional to magnetic flux quantum

$$\frac{dN}{dp_y} = \frac{L_y}{2\pi\hbar}$$

$$0 < \frac{p_y c}{eB} < L_x$$

$$0 < p_y < p_{y,\max} = \frac{eBL_x}{c}$$

$$N = \frac{dN}{dp_y} p_{y,\max} = \frac{eBL_y L_x}{2\pi\hbar c}$$

$$m = \frac{2\pi\hbar c n}{eB}$$

$$B = \frac{2\pi\hbar n}{\nu e} = \frac{n}{\nu}\Phi_0$$

# Conductivity in Quantum Mechanics

- Looking for ground state conductivity:
- We can define current in an intuitive way for a many body system
- Consider each direction of the current independently
- $I_x$ is exactly zero, since the momentum expectation value of the ground state harmonic oscillator is zero
- Since this is a shifted harmonic oscillator, we can consider each term individually.
- All terms dependent on wavenumber cancel

$$\mathbf{I} = -\frac{e}{m} \sum_{\text{filled states}} \langle \psi | -i\hbar\nabla + e\mathbf{A} | \psi \rangle$$

$$I_x = -\frac{e}{m} \sum_{n=1}^{\nu} \sum_{k} \langle \psi_{n,k} | -i\hbar \frac{\partial}{\partial x} | \psi_{n,k} \rangle = 0$$

$$I_y = -\frac{e}{m} \sum_{n=1}^{\nu} \sum_{k} \langle \psi_{n,k} | -i\hbar \frac{\partial}{\partial y} + exB | \psi_{n,k} \rangle$$

$$= -\frac{e}{m} \sum_{n=1}^{\nu} \sum_{k} \langle \psi_{n,k} | \hbar k + eBx | \psi_{n,k} \rangle$$

$$\langle \psi_{n,k} | x | \psi_{n,k} \rangle = -\hbar k / eB - mE/eB^2$$

# Quantum Conductivity pt. 2

- Can use definition of N to obtain J from I
- Summing over k gives us N because Fermions
- Using Ohm's Law (Obtained Earlier) we can find the conductance and resistivity of this quantum system
- Magically, it matches what we experimentally found to be the case!
- Used as the SI definition of value of magnetic flux quanta

$$I_y = e\nu \sum_k \frac{E}{B}$$

$$\mathbf{E} = \begin{pmatrix} E \\ 0 \end{pmatrix} \quad \Rightarrow \quad \mathbf{J} = \begin{pmatrix} 0 \\ e\nu E/\Phi_0 \end{pmatrix}$$

Any Questions?