Quiz #7

Friday, October 13

PHYSICS 851, FALL 2000

1. Describe the differences between

   (a) Spherical Bessel functions $j_\ell(x)$,

   (b) Spherical Neumann functions $n_\ell(x)$.

   (c) Hankel functions $h_\ell(x)$

   (d) $h_\ell^*(x)$.

2. Consider the Schrödinger equation for the function $u_\ell(r) = rR_\ell(r)$,

$$-\frac{d^2}{dr^2}u_\ell(r) + \frac{\ell(\ell+1)}{r^2}u_\ell(r) = k^2 u_\ell(r).$$

   Show that if I write the wave function at small r as

$$u_\ell \sim r^\alpha \left(A_0 + A_1 r + A_2 r^2 + \cdots\right),$$

   plug it into the Schrödinger equation above, and expand in r, that

   (a) $\alpha = \ell + 1, \quad \text{or} \quad -\ell$

   (b) $A_1 = 0$

3. If a particle of mass m experiences a spherical potential

$$V(r) = \frac{\beta}{r^2},$$

   describe the behavior of $u_\ell(r)$ at small r, i.e. if $u_\ell(r) \sim r^\alpha$, what is α?