your name(s)____

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Physics 852 Quiz #10 - Friday, Jan. 24th

The ω meson (mass=782 MeV) is charge neutral and has total isospin $I = 0$. For reasons we won't explain (g-parity) it mainly decays to a 3-pion channel. The pions, π^+, π^0, π^- are an I=1 isotriplet. If you couple the three isospins together, the projections of the pion's isospin, m_1, m_2, m_3, couple to total isospin I and I_{12} and projection M.

PART I.

1. What values of I_{12} are allowed?
2. Write the isospin portion of the ω wave function for pions with final momenta $\vec{k}_1, \vec{k}_2, \vec{k}_3$ as a sum of products of terms of the form, e.g. $\pi_1^+ \pi_2^- \pi_3^0$.
3. What are the branching ratios to various combinations of m_1, m_2, m_3 for the ω decay?

PART II.

You can re-write the pion states as π_x, π_y, π_z defined by

$$\begin{aligned}
\pi_z &= \pi_0 \\
\pi_x &= \frac{1}{\sqrt{2}}(\pi^+ + \pi^-), \\
\pi_y &= \frac{-i}{\sqrt{2}}(\pi^+ - \pi^-).
\end{aligned}$$

1. Using your Clebsch-Gordan skills write

$$S = \vec{\pi}_1 \cdot \vec{\pi}_2 = \pi_{1,x}\pi_{2,x} + \pi_{1,y}\pi_{2,y} + \pi_{1,z}\pi_{2,z}$$

   in terms of $\pi_i^{+,0,-}$ operators.
2. In the Cartesian basis, using $\vec{\pi}_1, \vec{\pi}_2, \vec{\pi}_3$, write an expression involving all three labels (1,2,3) with pion fields to the 3$^{\rm rd}$ order that is an isoscalar.
3. Rewrite this in terms of the $\pi^{+,0,-}$ basis.

PART I

1. $I_{12} = 1$, so that it can couple with pions ($I=1$) to isoscalar

2. $$|\omega\rangle = \sum_{M_{12}, M_3} C^{11}_{m_{12} m_3; 00} \, |I_{12}=1, I_\pi=1, M_{12}, M_3\rangle$$

$$= \sum_{m_3} C^{11}_{-m_3 m_3; 00} \, |I_{12}=1, I_\pi=1, -m_3, m_3\rangle$$

$m_3 = 1$

$$= C^{11}_{-1,1;00} \frac{|\pi^0\pi^-\pi^+\rangle - |\pi^-\pi^0\pi^+\rangle}{\sqrt{2}}$$

$$+ C^{11}_{00;00} \frac{|\pi^+\pi^-\pi^0\rangle - |\pi^-\pi^+\pi^0\rangle}{\sqrt{2}}$$

$$+ C^{11}_{1,-1;00} \frac{|\pi^+\pi^0\pi^-\rangle - |\pi^0\pi^+\pi^-\rangle}{\sqrt{2}}$$

$$|I=2, M=0\rangle = \frac{1}{\sqrt{6}}\left(|\pi^+\pi^-\rangle + 2|\pi^0\pi^0\rangle + |\pi^-\pi^+\rangle\right)$$

$$|I=1, M=0\rangle = \frac{|\pi^+\pi^-\rangle - |\pi^-\pi^+\rangle}{\sqrt{2}}$$

$$|I=0, M=0\rangle = \frac{|\pi^+\pi^-\rangle - |\pi^0\pi^0\rangle + |\pi^-\pi^+\rangle}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{6}}\left\{ |\pi^0\pi^-\pi^+\rangle - |\pi^-\pi^0\pi^+\rangle - |\pi^+\pi^-\pi^0\rangle + |\pi^-\pi^+\pi^0\rangle + |\pi^+\pi^0\pi^-\rangle - |\pi^0\pi^+\pi^-\rangle \right\}$$

100 % to $\pi^0\pi^-\pi^+$ never to $\pi^0\pi^0\pi^0$

PART II

(1)
$$\pi^+ = (\pi_x + i\pi_y)/\sqrt{2}$$
$$\pi^- = (\pi_x - i\pi_y)/\sqrt{2}$$
$$\pi^0 = \pi_z$$
$$\pi_x^2 + \pi_y^2 + \pi_z^2 = \pi_0^2 + \left(\frac{\pi_+ + \pi_-}{\sqrt{2}}\right)^2 + \left(\frac{\pi^+ - \pi^-}{i\sqrt{2}}\right)^2$$
$$= \pi_0^2 + \pi^+\pi^- + \pi^-\pi^+$$

(2)
$$w = (\vec{\pi}_1 \times \vec{\pi}_2)\cdot\pi_3$$
$$= \pi_x\pi_y\pi_z - \pi_y\pi_x\pi_z$$
$$+ \pi_y\pi_z\pi_x - \pi_z\pi_y\pi_x$$
$$+ \pi_z\pi_x\pi_y - \pi_x\pi_z\pi_y$$

(3)
$$= \frac{-i}{2}\Big\{(\pi_+ + \pi_-)(\pi^+ - \pi^-)\pi^0 - (\pi^+ - \pi^-)(\pi_+ + \pi_-)\pi^0$$
$$+ (\pi_+ - \pi_-)\pi_0(\pi_+ + \pi_-) - \pi_0(\pi_+ - \pi_-)(\pi_+ + \pi_-)$$
$$+ \pi_0(\pi_+ + \pi_-)(\pi_+ - \pi_-) - (\pi_+ + \pi_-)\pi_0(\pi_+ - \pi_-)\Big\}$$

$$= \frac{-i}{2}\Big\{\cancel{\pi^+\pi^+\pi^0(1-1)} + \cancel{\pi^-\pi^-\pi^0(-1+1)}$$
$$+ \pi^+\pi^-\pi^0(-1-1) + \pi^-\pi^+\pi^0(1+1)$$
$$+ \pi^+\pi^0\pi^-(1+1) + \pi^-\pi^0\pi^+(-1-1)$$
$$+ \pi^0\pi^+\pi^-(-1-1) + \pi^0\pi^-\pi^+(1+1)\Big\}$$

$$= -i\Big\{-|\pi^+\pi^-\pi^0\rangle + |\pi^-\pi^+\pi^0\rangle + |\pi^+\pi^0\pi^-\rangle - |\pi^-\pi^0\pi^+\rangle$$
$$- |\pi^0\pi^+\pi^-\rangle + |\pi^0\pi^-\pi^+\rangle\Big\}$$

SAME AS I !

I.2 & II.3 have same answer within constant. Result has "mixed" symmetry. → Anti-symmetric w.r.t $1 \Leftrightarrow 2$, but neither symmetric nor anti-symmetric w.r.t. $1 \Leftrightarrow 3$ or $2 \longleftrightarrow 3$.

Matrix element would have form

$\longleftrightarrow (\vec{k}_1 - \vec{k}_2) \cdot \vec{z\omega}$. This would make all the elements symmetric.