your name(s)____

____

Physics 852 Quiz #13 - Friday, Feb. 14th

In the Sommerfeld model, electrons from the conduction band are primarily treated as if they are in a non-interacting Fermi gas, i.e. they interact neither with the lattice or the core electrons, nor with one another. However, when calculating the conductivity they can collide with one another. Here, we consider Aluminum, which has three electrons per atom in the conduction band. The mass density of aluminum is 2.70 g/cm^3, and the mass of an aluminum atom is 26.98 u, where the mass of 1 u is 1.67×10^{-24} g.

1. What is the Fermi wave vector in Å^{-1}?
2. What is the Fermi energy in eV?
3. The intrinsic spin of the electron couples to a magnetic field to give an energy,

$$H_B = g_s \frac{e\hbar}{2mc} \frac{\vec{s}}{\hbar} \cdot \vec{B},$$

in c.g.s. units where $\vec{s}$ is the spin of the electron. Note that the magnetic coupling is $-\vec{\mu} \cdot \vec{B}$, but because the electron has a negative charge, $\vec{\mu} = -g_s e/(2mc)\vec{s}$, so the sign of H_B is positive. The $g-$factor for the electron is approximately 2.0. The Bohr magneton is

$$\mu_B = \frac{e\hbar}{2mc}.$$

If an electron is subjected to a 3 Tesla field, what is energy difference between a spin-up and a spin-down electron? in eV. Note: This is a unit-conversion nightmare.

4. Derive an expression for the magnetic susceptibility defined by

$$\chi_V = \frac{dM}{dB},$$
$$\vec{M} = \frac{1}{V} \sum_i \vec{\mu}_i,$$
$$\vec{\mu}_i = -g_s \mu_B \frac{\vec{s}_i}{\hbar}.$$

Here, $\vec{m}$ is the magnetic moment per unit volume, and is proportional to the difference in the densities of spin-up and spin-down electrons. Base your calculation by assuming that the Fermi energies for the spin-up and spin-down electrons adjust themselves to minimize the overall energy.

5. The magnetic susceptibility is dimensionless, and the measurment for Aluminum is $1.75 \times$ ~~[illegible]~~ $\times 10^{-6}$. How does your answer compare?

1. $\rho_m = 2.70\ g/cm^3$, $m_p = 1.67 \cdot 10^{-24} g$

$m_a = m_p \cdot 26.98$

3 electrons / atom

$n = 0.1798\ \mathring{A}^{-3}$

$= \frac{2}{6\pi^2} k_f^3$, $k_f = 1.746\ \mathring{A}^{-1}$

2. $\varepsilon_F = \frac{\hbar^2 k_f^2}{2m} = 11.62\ eV$

3. $\Delta E = g_s \mu_B B$

$g_s = 2$, $B = 3$

$\mu_B = 9.274 \cdot 10^{-24}\ J\,T^{-1}$

$= 0.347\ meV$

4. $\mu B = \Delta E_f = \frac{\hbar^2}{m} k_f \Delta k_f$

$\Delta \rho = \frac{3}{6\pi^2} k_f^2 \Delta k_f = \rho_\uparrow - \rho_\downarrow$

$M = g_s \mu_B \frac{1}{2} \Delta \rho$

$= g_s \frac{e\hbar}{4mc} \cdot \frac{1}{2\pi^2} k_f^2 \Delta k_f$

$= g_s \frac{e\hbar}{8\pi^2 mc} k_f \frac{m}{\hbar^2} g_s \mu_B B$

$= \frac{g^2 e^2 k_f}{16\pi^2 mc^2} B$

cgs $\rightarrow$ $\chi = \frac{1}{4\pi^2} \frac{e^2 k_f}{mc^2} = 1.25 \cdot 10^{-6}$

SI $\chi_{SI} = 4\pi \cdot \chi_{cgs} = 1.57 \cdot 10^{-5}$

$\chi_{exp} = 1.75 \cdot 10^{-6}$