your name(s)____

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Physics 852 Quiz #14 - Friday, Feb. 21st

Imagine you have calculated the matrix element

$$I_0 = \langle \alpha_f, \ell_f = 1, m_f = 0 | P_z^2 | \alpha_i, \ell_i = 3, m_i = 0 \rangle.$$

1. (10 pts) For all m_i, m_f, express the elements $\langle \alpha_f, \ell = 1, m_f | P_z^2 | \alpha_i, \ell = 3, m_i \rangle$ in terms of I_0 and Clebsch-Gordan coefficients.
2. (10 pts) For all m_i, m_f, express the elements $\langle \alpha_f, \ell = 1, m_f | (P_x^2 + P_y^2) | \alpha_i, \ell = 3, m_i \rangle$ in terms of I_0 and Clebsch-Gordan coefficients.
3. (10 pts) For all m_i, m_f, express the elements $\langle \alpha_f, \ell = 1, m_f | (P_x^2 - P_y^2) | \alpha_i, \ell = 3, m_i \rangle$ in terms of I_0 and Clebsch-Gordan coefficients.

$$T^2_0 = \frac{1}{2}(3z^2 - r^2) = \frac{1}{2}(2r^2 - x^2 - y^2)$$

$$T^2_{\pm 2} = \sqrt{\frac{3}{8}}(x^2 - 2ixy - y^2)$$

$$z^2 = \frac{2}{3} T^2_0$$

$$(x^2 + y^2) = -2T^2_0$$

$$(x^2 - y^2) = \sqrt{\frac{2}{3}}(T^2_{-2} + T^2_2)$$

1. $\langle \alpha_f, \ell = 1, m_f | P_z^2 | \alpha_i, \ell = 3, m_i \rangle$

=

m_i	m_f	
1	1	$I_0 \, C^{2\,3}_{0\,1;11} / C^{2\,3}_{0\,1;11}$
0	0	$I_0 \, \cancel{C^{2\,3}_{0\,0;10} / C^{2\,3}_{0\,0;10}}$
−1	−1	$I_0 \, C^{2\,3}_{0\,-1;1-1} / C^{2\,3}_{0\,-1;1-1}$

2. $\langle m_f | P_x^2 + P_y^2 | m_i \rangle$

$$= \frac{3}{2} \cdot (-2)\, I_0 \cdot \frac{C^{2\,3}_{0\,m_i;\,1\,m_i}}{C^{2\,3}_{0\,m_f;\,1\,m_f}}$$

$\neq 0$ for

$m_i = m_f = 1$

$m_i = m_f = 0$

$m_i = m_f = -1$

3. $\langle m_f | P_x^2 - P_y^2 | m_f \rangle$

$$= \frac{3}{2}\sqrt{\frac{2}{3}}\, I_0 \left(\frac{C^{2\,3}_{2\,m_i;\,1\,m_f}}{C^{2\,3}_{0\,0;\,1\,0}} + \frac{C^{2\,3}_{-2\,m_i;\,1\,m_f}}{C^{2\,3}_{0\,0;\,1\,0}} \right)$$

$\neq 0$ for

$m_i = 3,\ m_f = 1$

$m_i = 2,\ m_f = 0$

$m_i = 1,\ m_f = -1$

$m_i = -1,\ m_f = 1$

$m_i = -2,\ m_f = 0$

$m_i = -3,\ m_f = 1$