

your name(s) \_\_\_\_\_

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Physics 852 Quiz #15 - Friday, March. 13th

BCS theory is built on a simplified assumption of an interaction with a fixed density of states over a finite energy range, with an interaction that mixes all states (with a given total momentum) equally. To mimic this, consider the following Hamiltonian in matrix form,

$$\begin{aligned} H_{ii} &= E_0(i^2/M^2), & E_0 &= 1.0 \\ H_{i \neq j} &= -v_0/\sqrt{M}, & V_0 &= 0.1; \end{aligned} \quad (0.1)$$

where  $M$  is the dimensionality of the matrix with  $0 \leq i < M$ .

1. Write a program that finds the eigenvalues of the matrix and prints out the lowest 10 eigenvalues.
2. Run the program for  $M = 5, 20, 80, 320$  and compare results.
3. For  $M = 320$ , compare the result for  $v_0 = 0.1$  to results for  $v_0 = 0.001, 0.01$ .

**Solutions:**

Energies for  $v_0 = 0.001$

5	20	80	320
-0.000007	-0.000029	-0.000067	-0.000067
0.040002	-0.000007	-0.000029	-0.000052
0.160001	0.002504	-0.000007	-0.000034
0.360001	0.010002	0.000141	-0.000029
0.640002	0.022501	0.000620	-0.000007
99.999900	0.040001	0.001404	0.000003
99.999900	0.040002	0.002499	0.000058
99.999900	0.062501	0.002504	0.000131
99.999900	0.090001	0.003906	0.000141
99.999900	0.122501	0.005625	0.000223

Energies for  $v_0 = 0.01$

5	20	80	320
-0.000656	-0.001454	-0.001454	-0.001454
0.040185	-0.000656	-0.001055	-0.001055
0.160142	0.002094	-0.000754	-0.000754
0.360146	0.009895	-0.000656	-0.000656
0.640183	0.022499	-0.000159	-0.000555
99.999900	0.040043	0.000733	-0.000535
99.999900	0.040185	0.001923	-0.000496
99.999900	0.062567	0.002094	-0.000438
99.999900	0.090081	0.003413	-0.000360
99.999900	0.122590	0.005205	-0.000263

Energies for  $v_0 = 0.1$

5	20	80	320
-0.030831	-0.030831	-0.030831	-0.030831
0.032559	-0.021335	-0.021335	-0.021335
0.163912	-0.016383	-0.016383	-0.016383
0.371284	-0.006614	-0.011115	-0.011115
0.663077	0.008032	-0.010798	-0.010798
99.999900	0.027565	-0.010173	-0.010173
99.999900	0.032559	-0.009236	-0.009236
99.999900	0.051993	-0.007987	-0.007987
99.999900	0.081325	-0.006614	-0.006614
99.999900	0.115573	-0.006425	-0.006425