your name(s)____

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Physics 851 Quiz #3 - Friday, Oct. 4th

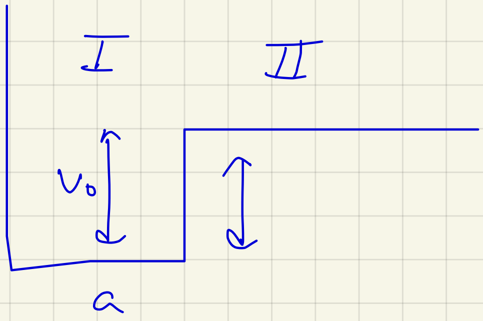
FYI: $\hbar c = 197.327$ MeV fm.

A beam of protons, of mass $m = 938.28$ MeV, are aimed (coming from $x = +\infty$), at the following potential.

$$V(r) = \begin{cases} \infty, & r < 0 \\ -V_0, & 0 < r < a \\ 0, & r > a \end{cases},$$

where $a = 1.0$ fm.

1. (5 pts) Find the minimum value of V_0 (in MeV), for having a bound state. Refer to this value as $V_{0,\mathrm{min}}$.
2. (5 pts) The phase shift δ for scattered waves is such that the solution for $r > a$ is
$$\begin{aligned} \psi_p(r) &\sim e^{-ipr/\hbar} - e^{ipr/\hbar + 2i\delta(p)} \\ &= -2ie^{i\delta} \sin(pr/\hbar + \delta). \end{aligned}$$
Calculate the phase shift as a function of the momentum p. Note, the wave function has the form $\sin(pr/\hbar + \delta)$ for $r > a$ and the form $A\sin(p'r/\hbar)$ for $r < a$.
3. (5 pts) As a function of p plot the phase shift for $V_0 = 1.01V_{0,\mathrm{min}}$ and for $V_0 = 0.99V_{0,\mathrm{min}}$. The phase shifts should be between zero and 180 degrees, and make the plots for $0 < p < 600$ MeV/c.
4. (5 pts) Now assume the system is further contained by a very large box of length L, i.e, $V(r > L) = \infty$. As a function of p, L, and δ, give the equation that determines what values of p are allowed?
5. (5 pts) In terms of p, L and δ what is the number of states per unit p, dN_{states}/dp?
6. (5 pts) Integrating the density of states over all momenta, how many EXTRA states are introduced by the fact that $V_0 \neq 0$? Give answers for both values of V_0 above. Note, for finite potentials phase shifts go to zero as $p \to \infty$.



① $ka = \pi/2$

$$V_0 = \frac{\hbar^2 k^2}{2M} = \frac{\hbar^2 \pi^2}{8Ma^2}$$

② $\psi_I = A \sin qr$

$$\psi_{II} = \sin(kr + \delta)$$

$$A \sin qa = \sin(ka + \delta)$$

$$qA \cos qa = k \cos(ka + \delta)$$

$$\frac{1}{q} \tan qa = \frac{1}{k} \tan(ka + \delta)$$

$$\delta = -ka + \tan^{-1}\left(\frac{k}{q} \tan qa\right)$$

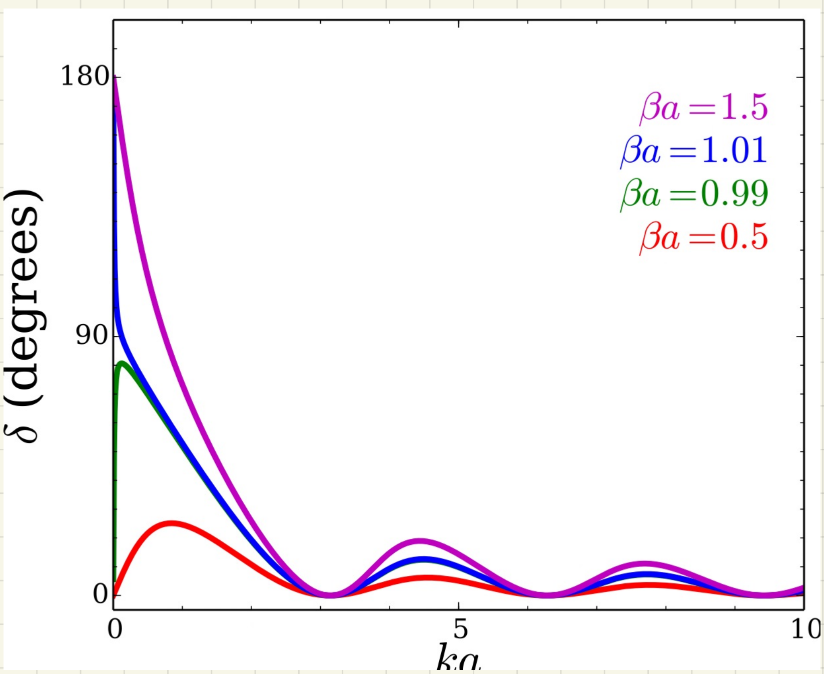
$$\frac{\hbar^2 k^2}{2M} = E$$

$$k = \sqrt{2mE/\hbar^2}$$

$$\frac{\hbar^2 q^2}{2M} = E + V_0$$

$$q = \sqrt{2m(E+V_0)/\hbar^2}$$

③



(4) $\sin(kL+\delta) = 0$

$$kL + \delta = n\pi$$

(5)
$$\frac{dN_{\text{states}}}{dp} = \frac{dn}{\hbar\, dk} = \frac{1}{\hbar}\left(\frac{L}{\pi} + \frac{1}{\pi}\frac{d\delta}{dk}\right)$$

$$= \frac{L}{\pi\hbar} + \underbrace{\frac{1}{\pi}\frac{d\delta}{dp}}_{\text{extra states}}$$

(6)
$$\Delta N = \int_0^\infty dp\, \frac{1}{\pi}\frac{d\delta}{dp} = \frac{-\delta(0)}{\pi} + \underbrace{\frac{\delta(p=\infty)}{\pi}}_{=0}$$

due to interaction

$$= 0 \quad \text{for } V_0 = 0.99\,\frac{\hbar^2\pi^2}{8ma^2}$$

$$= -1 \quad \text{for } V_0 = 1.01\,\frac{\hbar^2\pi^2}{8ma^2}$$

state lost from continuum to form bound state