your name(s)________________________________

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Physics 851 Quiz #4 - Friday, Oct. 18th

Imagine a system where you have t particles with angular momentum $\ell = 1$. You want to count how many ways you can combine the particles to have some total momentum L. If the number of multiplets of type L for t particles is $N(L,t)$,

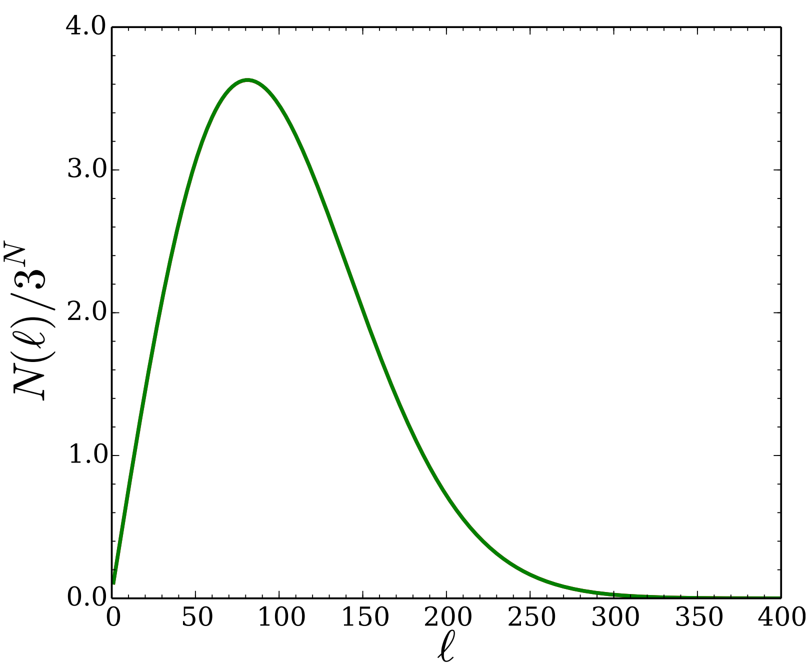
$$\begin{aligned} N(L,t+1) &= N(L-1,t) + N(L,t) + N(L+1,t), \quad L > 0, \\ N(0,t+1) &= N(1,t). \end{aligned}$$

Because the number of multiplets rises as 3^t, we consider the quantity $\rho \equiv N/3^t$. The equations for ρ are

$$\begin{aligned} \rho(L,t+1) &= \frac{\rho(L-1,t) + \rho(L,t) + \rho(L+1,t)}{3}, \quad L > 0, \\ \rho(0,t+1) &= \rho(1,t)/3. \end{aligned}$$

1. (5 pts) Treating the step sizes, $\Delta t = 1$, and $\Delta L = 1$ as small, write a differential equation for ρ, i.e.,
$$\frac{d}{dt}\rho(L,t) = \cdots$$
2. Find a solution for the differential equation with the condition $\rho(L, t = 0) = \delta(L)$.
3. Find a solution for the differential equation with the boundary condition that $\rho(L = -1/2) = 0$, and that $\rho(L, t = 0) = \delta(L)$. Hint, use the method of images.
4. Express the difference of the two contributions in (3) as a derivative w.r.t. L, i.e. treat the difference between the two sources as a small number. This should be accurate at large times when the scales over which the distribution changes is much larger than unity.
5. Plot this solution for $t = 10000$.

The true solution, from using the recurrence relations above, is plotted below.



1. $$\rho(L, t+1) = \left[\rho(L-1, t) + \rho(L, t) + \rho(L+1, t)\right]\frac{1}{3}$$

$$\rho(L, t+1) - \rho(L, t) = \frac{1}{3}\left\{\rho(L-1, t) - 2\rho(L, t) + \rho(L+1, t)\right\}$$

$$\frac{d\rho}{dt} = \frac{1}{3}\frac{d^2\rho}{dL^2} \qquad D = \frac{1}{3}$$

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2. $$\rho(L, t) = \frac{1}{\sqrt{4\pi D t}} \exp\left\{-\frac{L^2}{4Dt}\right\}$$

$$\partial_t \rho = -\frac{1}{2t}\rho + \frac{L^2}{4Dt^2}\rho$$

$$\partial_L \rho = \left(-\frac{L}{2Dt}\right)\rho$$

$$\partial_L^2 \rho = -\frac{1}{2Dt}\rho + \frac{L^2}{4D^2t^2}\rho = \frac{1}{D}\frac{\partial \rho}{\partial t} \quad \checkmark$$

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3. $$\rho = \frac{1}{\sqrt{4\pi D t}}\left\{e^{-L^2/4Dt} - e^{-(L+1)^2/4Dt}\right\}$$

for $L > -\frac{1}{2}$

4. $$= \Delta L \frac{d}{dL} \frac{-1}{\sqrt{4\pi D t}} e^{-(L+\frac{1}{2})^2/4Dt} \qquad \Delta L = 1$$

$$= \frac{2\pi L}{(4\pi D t)^{3/2}} e^{-(L+\frac{1}{2})^2/4Dt}$$

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5. Looks just like plot on previous page.