your name(s)_____

_____

*Physics 851 Quiz #5 - Friday, Oct. 25th*

Consider a particle of mass $M$ confined to a two-dimensional circle of radius $R$. The particle moves in a periodic potential,

$$V(\phi + 2\pi/N) = V(\phi),$$

where $N$ is an integer. Assuming the wave-function has the form,

$$\psi(\phi) = e^{im\phi} + Be^{-im\phi}, \;\; 0 < \phi < 2\pi/N,$$

and that the eigenstate of the translation operator, $\mathcal{R}(2\pi/N) = e^{i\gamma}$, i.e.,

$$\psi(\phi + 2\pi/N) = e^{i\gamma}\psi(\phi).$$

In your homework you showed that $m$ could be found that the allowed values of $\gamma$ were $j\alpha$, where $\alpha = 2\pi/N$. You then considered a potential of the form,

$$V(\phi) = \sum_{j=1,N} \delta(\phi - 2\pi j/N),$$

and found a transcendental equation for $m$,

$$\begin{aligned} 0 &= p\sin(m\alpha) + 2m\cos(m\alpha) - 2m\cos(j\alpha), \\ \gamma &= j\alpha, \; \alpha = 2\pi/N, \\ p &= 2M\beta R^2/\hbar^2. \end{aligned}$$

Write a program (you can use any packages you wish, to solve for the lowest 4 values of $m$ as a function of $\gamma$. Plot $m$ (energy is $\hbar^2 m^2/2MR^2$) as a function of $\gamma$ for $-\pi \leq \gamma \leq \pi$. Assume $p\alpha = 5.0$ and make plots for $N = 4$ and for $N = 100$. Note that the function will be multi-valued because you will find four values of $m$ for each $\gamma$.