

your name(s) \_\_\_\_\_

---

*Physics 851 Quiz #8 - Wednesday, Nov 27th*

Consider the one-dimensional situation, with a particle of mass  $m = 938.38 \text{ MeV}/c^2$ , in a potential

$$V(x) = \begin{cases} \infty, & x < 0 \\ V_0 e^{-x^2/2R^2}, & x > 0 \end{cases}$$

$V_0 = 12 \text{ MeV}$ ,  $R = 3 \text{ fm}$  or  $30 \text{ fm}$

A particle with energy  $E = 15 \text{ MeV}$  is incident on the potential from  $x = +\infty$  and reflect backward. The wave function for large  $x$  has the form

$$\psi(x) = (e^{2i\delta} e^{ikx} - e^{-ikx})/2$$

1. Write a program that solves for  $|\psi(x)|^2$ , then plots it for  $0 < x < 4R$ . Note  $\hbar c = 197.326 \text{ MeV fm}$ .
2. Using energy conservation, use classical physics to solve for the momentum of the particle as a function of  $x$ ,  $p(x, p_\infty)$ , where  $p_\infty$  is the asymptotic momentum. On the same graph as above, plot  $dp/dp_\infty$  as a function of  $x$ .

If you wish you can use the following python code to solve for the wave function,

```
def V(VV0,R,xx):
    return VV0*exp(-0.5*xx*xx/(R*R))

hbarc=197.326
N=2400
m=939.0
V0=12.0
E=15.0
R=float(input("Enter R: "))
Rmax=8*R
x=np.ndarray(shape=(N),dtype=float)
dx=Rmax/N
psi=np.ndarray(shape=(N),dtype=complex)
psisquared=np.ndarray(shape=(N),dtype=float)
ci=complex(0,1.0)
x[N-1]=(N-1)*dx
x[N-2]=(N-2)*dx
k=sqrt(2.0*m*E)/hbarc
psi[N-1]=exp(-ci*k*x[N-1])
psi[N-2]=exp(-ci*k*x[N-2])
for i in range(N-3,-1,-1):
    x[i]=i*dx
    psi[i]=2.0*psi[i+1]-psi[i+2]-2.0*m*dx*dx*(E-V(V0,R,x[i+1]))*psi[i+1]/(hbarc*hbarc)

delta=-0.5*ci*log(psi[0]/conjugate(psi[0])).real
print('delta=',delta*180/pi)

for i in range(0,N):
    psi[i]=(-psi[i]+exp(2.0*ci*delta)*conjugate(psi[i]))/(2.0*ci)
    psisquared[i]=(psi[i]*conjugate(psi[i])).real

plt.plot(x,psisquared,linestyle='-',linewidth=2,color='r')
```