your name(s)____

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Physics 852 Quiz #2 - Friday, Jan. 17th

Consider two kinds of spinless particles, whose masses are m_A and m_B. The particles exist in a one-dimensional world, and if one defines field operators,

$$\Phi_a(x) = \sum_k \frac{1}{\sqrt{LE_a(k)}}\left(a_k e^{ikx} + a_k^\dagger e^{-ikx}\right),$$
$$\Phi_b(x) = \sum_k \frac{1}{\sqrt{LE_b(k)}}\left(b_k e^{ikx} + b_k^\dagger e^{-ikx}\right),$$

the interaction Hamiltonian is

$$H_{\rm int} = g\int dx\; \Phi_a(x)\Phi_b(x)^2. \tag{0.1}$$

Now, let $m_A > 2m_B$, so that the heavier particle can decay into two lighter particles. Also assume the decay energy is sufficiently high that the lighter particles move relativistically, $E_b^2 = \hbar^2 k_b^2 + m_b^2$. Calculate the decay rate, Γ, for the reaction $A \to 2B$ in lowest order perturbation theory. Express your answer in terms of m_A, m_B and the coupling constant g. Hint: assume the decaying particle is in a zero momentum state, then use the integral over x to enforce conservation of momentum of the two outgoing particles. Note that the momentum states are orthogonal,

$$\int dx\; e^{ik_a x}e^{ik_b x} = L\delta_{k_a,k_b}.$$

$$\langle f | H_{int} | i\rangle = \langle 0 | b_k b_{k'} H_{int}\, a^\dagger_{k=0} | 0\rangle$$

$$= \int dx\, e^{i(k+k')x} \frac{2g}{L^{3/2}(E_b(k)E_b(k')M_a)^{1/2}}$$

factor accounts for 2 ways link $b^\dagger_1, b^\dagger_2$ in Φ_b^2 to $b_k\, b_{k'}$

$$= L\,\delta_{k,-k'} \frac{2g}{L^{3/2} E_b M_a^{1/2}}$$

$$\Gamma = \frac{2\pi}{\hbar}\sum_k{}' |\langle f|H_{int}|i\rangle|^2 \cdot \delta(\mathcal{E}_f - \mathcal{E}_i) = \frac{2\pi}{\hbar}\int_0^\infty \frac{dk}{2\pi} L\,|\langle\ \ \rangle|^2 \cdot \delta(\mathcal{E}_f - \mathcal{E}_i)$$

don't double count for ident parts!

$$= \frac{4g^2}{\hbar M_a}\int \frac{dk}{E_b^2}\,\delta(2E_b - M_a) = \frac{4g^2}{\hbar m_a}\frac{1}{E_b^2}\frac{1}{2\frac{dE_b}{dk}}$$

$$= \frac{4g^2}{(\hbar^2 m_a^3/4)}\frac{\frac{1}{2}(M_a/2)}{\sqrt{\left(\frac{M_A}{2}\right)^2 - M_b^2}} = \frac{4g^2}{\hbar^2 M_A^2\sqrt{\left(\frac{M_A}{2}\right)^2 - M_B^2}}$$