Physics 853 Quiz #3 - Monday, Sep. 27, 2010

1. Consider the matrix

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = -i\alpha_x\alpha_y\alpha_z.$$

   (a) Find the eigenvalues of γ_5.

   (b) Show that $(1+\gamma_5)/2$ is a projection operator.

   (c) Write down the matrix components of γ_5 in the chiral representation, i.e. where

$$\vec{\alpha} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & -\vec{\sigma} \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}.$$

   FYI: $\epsilon_{ijk}\sigma_i\sigma_j = i\sigma_k$

   (d) Consider a positive-energy plane-wave solution with positive/negative helicity,

$$\begin{aligned} (\boldsymbol{p}\cdot\boldsymbol{\alpha})u_\pm(\boldsymbol{p}) &= E_p u_\pm(\boldsymbol{p}), \\ (\boldsymbol{\Sigma}\cdot\boldsymbol{p})u_\pm(\boldsymbol{p}) &= \pm E_p u_\pm(\boldsymbol{p}), \end{aligned}$$

   where the spin operator $\Sigma_k = -i\epsilon_{ijk}\alpha_i\alpha_j/2$. Independent of the representation, show that for such solutions,

$$\gamma_5 u_\pm(\boldsymbol{p}) = \pm u_\pm(\boldsymbol{p}).$$

   HINT: Consider $\{\boldsymbol{p}\cdot\boldsymbol{\alpha}, \boldsymbol{p}\cdot\boldsymbol{\Sigma}\}u_\pm(\boldsymbol{p})$, and you may appreciate the identity $\{\alpha^m, \epsilon_{ijk}\alpha^j\alpha^k\} = 2\delta_{im}\epsilon_{ijk}\alpha^i\alpha^j\alpha^k$ (note: don't sum over i).