*Physics 853 Quiz #5 - Monday, October. 18, 2010*

Here, you will solve for the pair creation density for a one-dimensional complex scalar field undergoing a sudden change of mass using Bogoliubov transformations. Consider the mass to be zero for $t < 0$, but equal to $M$ for $t > 0$.First, consider the scalar field

$$\begin{aligned}\Phi(x, t < 0) &= \int \frac{dk}{(2\pi)2E_k} e^{ikx} \left[a(k)e^{-iE_k t} + b^\dagger(-k)e^{iE_k t}\right],\\ \Phi(x, t => 0) &= \int \frac{dk}{(2\pi)2E'_k} e^{ikx} \left[a'(k)e^{-iE'_k t} + b'^\dagger(-k)e^{iE'_k t}\right]\end{aligned}$$

The energies are $E_k = |k|, E'_k = \sqrt{k^2 + M^2}$. The creation and destruction operators will be normalized such that,

$$[a(k), a^\dagger(q)] = 2E_k(2\pi)\delta(k - q).$$

1. What are the boundary conditions relating $\Phi(x, t = 0^-)$ to $\Phi(x, t = 0^+)$? HINT: Remember that in the Klein-Gordon equation, there are terms that go as $\partial_t^2$, and that the charge density goes as $i(\Phi^*\dot{\Phi} - \dot{\Phi}^*\Phi)/2$.

2. Express $a(k)$ as an integral over $dx$ with the arguments involving $\Phi(x, t = 0)$ and $\dot{\Phi}(x, t = 0)$.

3. Using the BC mentioned above, find the coefficients, $\alpha(k, M)$ and $\beta(k, M)$, such that

$$a'(k) = \alpha a(k) + \beta b^\dagger(-k).$$

4. Find $dN_{\rm pairs}/(dkdL)$, where $L$ is the length of the system.