Physics 853 Quiz #6 - Monday, November 8, 2010

Spin 3/2 particles are also fermions. The way in which they are described in field theory is through operators, $\Psi^\mu(x)$. Like spin-1/2 particles, the wave functions have 4 spinor indices. Combined with the four Lorentz indices, μ, $\Psi(x)$ has effectively 16 indices. This is four times as many as spin 1/2 particles. For spin 1/2 particles the indices correspond to particle/anti-particle, and spin-up/spin-down. For spin 3/2 particles, one expects for $m_s = 3/2, 1/2, -1/2, -3/2$, and for particle anti-particles, to have 8 indices. Thus, Ψ^μ has double the number of indices as the needed number of degrees of freedom. This is accounted for by two constraints,

$$\partial_\mu \Psi^\mu = 0 \qquad \gamma_\mu \Psi^\mu = 0.$$

The 8 constraints above then project away the unwanted degrees of freedom. In momentum space, the propagator has the form,

$$\begin{aligned} G^{\alpha\beta}(k) &= \frac{P^{\alpha\beta}}{\not{p} - m + i\epsilon}, \\ P^{\alpha\beta} &= g^{\alpha\beta} + A\gamma^\alpha\gamma^\beta + B\gamma^\alpha \not{p} p^\beta/p^2 + Cp^\alpha \not{p}\gamma^\beta/p^2 + Dp^\alpha p^\beta/p^2, \end{aligned}$$

where $P^{\alpha\beta}$ is a projection operator, and A, B, C and D are scalar functions of p^2. (Note that each term has implicit spinor indices ij, e.g., $g^{\alpha\beta} \to g^{\alpha\beta}\delta_{ij}$).

1. (10 pts) Find A, B, C and D by applying the constraints above.
2. (5 pts) Show that $P^{\alpha\beta}$ is a projector.

Some info: $\gamma^\alpha\gamma_\alpha = 4$, $\not{p}^2 = p^2$.