

Physics 853 Quiz #4 - Monday, October 19, 2011

1. (10 points) Consider the coherent state,

$$|\eta\rangle = \frac{1}{Z^{1/2}} e^{\eta a^\dagger} |0\rangle.$$

- (a) Derive $Z(\eta)$ such that $\langle\eta|\eta\rangle = 1$.
(b) For a normalized state $|m\rangle$, where $a^\dagger a|m\rangle = m|m\rangle$, find $|\langle m|\eta\rangle|^2$, i.e., the probability m quanta would be observed.
(c) Explicitly calculate the sum

$$\sum_{m=0}^{\infty} |\langle m|\eta\rangle|^2.$$

- (d) Explicitly calculate the sum

$$\sum_{m=0}^{\infty} m |\langle m|\eta\rangle|^2.$$

2. (10 points Extra Credit) Consider the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\phi\partial^2\phi + \frac{1}{2}\psi\partial^2\psi - \frac{\mu^2}{4}(\phi - \psi)^2,$$

where ϕ and ψ are real scalar fields.

Express ϕ and ψ as field operators in the form of integrals over d^3p where the integrands involve operators that create and destroy eigenstates of the Hamiltonian, and also obey the constraint that $[\phi(\mathbf{r}, t), \pi(\mathbf{r}', t)] = i\delta^3(\mathbf{r} - \mathbf{r}')$.