Physics 853 Quiz #6 - Monday, November 9, 2011

Consider the conductivity tensor, defined by

$$J_i = \sigma_{ik} E_k.$$

1. Show that this can be expressed as

$$\sigma_{ij} = \frac{-ie}{\hbar} \int_{-\infty}^{0} dt' \int d^3x' x'_j \; \langle \Psi_0 | [J_i(0), \rho(\boldsymbol{x}', t')] | \Psi_0 \rangle,$$

and $|\Psi_0\rangle$ is the state in the absence of electric field. Your derivation should begin with the expression in first-order perturbation theory that:

$$|\Psi\rangle = |\Psi_0\rangle - \frac{i}{\hbar} \int_{-\infty}^{0} dt \; V(t) |\Psi_0\rangle.$$

2. Show that this can be expressed as:

$$\sigma_{ij} = \frac{-i}{\hbar} \int_{-\infty}^{0} dt' \int d^3x' \; t' \langle [J_i(0), J_j(\boldsymbol{x}', t')] \rangle$$

3. Consider 1-D case (no indices) where one defines the quantity,

$$g(t) \equiv \frac{-i}{\hbar} \int d^3r \; \langle [J(0), J(\mathbf{r}, t)] \rangle.$$

Assuming that all quantities are invariant under switching indices i and j, show that:

(a)

$$g(t) = g^*(t).$$

(b)

$$g(t) = -g(-t).$$

(c)

$$g^*(\omega) = -g(\omega).$$

(d)

$$\sigma = \lim_{\omega \to 0} \frac{\mathcal{I}m \; g(\omega)}{2\omega}.$$

For the Fourier transforms, use the definitions

$$g(t) = \int d\omega e^{-i\omega t} g(\omega)/(2\pi),$$

$$g(\omega) = \int dt e^{i\omega t} g(t)$$