Physics 853 Quiz #4 - Monday, October 24, 2012

1. (10 points) Consider the coherent state,

$$|\eta\rangle = \frac{1}{Z^{1/2}} e^{\eta a^\dagger} |0\rangle.$$

   (a) Derive $Z(\eta)$ such that $\langle \eta | \eta \rangle = 1$.

   (b) For a normalized state $|m\rangle$, where $a^\dagger a |m\rangle = m|m\rangle$, find $|\langle m|\eta\rangle|^2$, i.e., the probability m quanta would be observed.

   (c) Explicitly calculate the sum

$$\sum_{m=0}^{\infty} |\langle m|\eta\rangle|^2.$$

   (d) Explicitly calculate the sum

$$\sum_{m=0}^{\infty} m|\langle m|\eta\rangle|^2.$$

2. Consider the Lagrangian density for a real scalar field

$$\mathcal{L} = \frac{1}{2}(\partial^\alpha \phi)(\partial_\alpha \phi) - \frac{1}{2}m^2\phi^2.$$

Express ϕ as a field operator in the form of an integral over d^3p where the integrands involve operators that create and destroy eigenstates of the Hamiltonian, and also obey the constraint that $[\phi(\mathbf{r},t), \pi(\mathbf{r}',t)] = i\delta^3(\mathbf{r}-\mathbf{r}')$. Get all factors correct and be sure to define commutation rules for $a(\mathbf{p})$ and $a^\dagger(\mathbf{p})$.