Physics 853 Quiz #5 - Friday, November 2, 2012

OPEN NOTES – OPEN BOOK – OPEN MOUTH

Here, you will solve for the pair creation density for a one-dimensional fermionic scalar field undergoing a sudden change of mass using Bogoliubov transformations. Consider the mass to be zero for $t < 0$, but equal to M for $t > 0$.First, consider the field operators

$$\begin{aligned}\Psi(x, t < 0) &= \sum_s \int \frac{dk}{(2\pi)E_k} e^{ikx} \left[b_s(k) u_s(k) e^{-iE_k t} + d_s^\dagger(-k) v_s(-k) e^{iE_k t} \right], \\ \Psi(x, t => 0) &= \sum_s \int \frac{dk}{(2\pi)E'_k} e^{ikx} \left[b'_s(k) u'_s(k) e^{-iE'_k t} + d'^\dagger_s(-k) v'_s(-k) e^{iE'_k t} \right]\end{aligned}$$

The energies are $E_k = |k|, E'_k = \sqrt{k^2 + M^2}$. The creation and destruction operators will be normalized such that,

$$[b_s(k), b^\dagger_{s'}(q)] = 2\pi E_k \delta(k - q)\delta_{ss'},$$

and the spinors are normalized such that

$$u^\dagger_s(k) u_{s'}(k) = E_k \delta_{ss'}, \qquad u^\dagger_s(k) v_{s'}(-k) = 0.$$

1. What are the boundary conditions relating $\Psi(x, t = 0^-)$ to $\Psi(x, t = 0^+)$?
2. Express $b(k)$ as an integral over dx with the arguments involving $\Psi(x, t = 0)$ and $u_s(k)$.
3. Using the BC mentioned above, find the coefficients, $\alpha(k, M)$ and $\beta(k, M)$, such that

   $$b'_{s'}(k) = \sum_s \alpha_{s's}(k) b_s(k) + \beta_{s's}(k) d^\dagger_s(-k),$$

   leaving α and β in the form of the overlap of spinors.
4. Express the coefficients α and β in terms of k, M and E_k.
5. In terms of α and β find $dN_{\text{pairs}}/(dkdL)$, where L is the length of the system. HINT: The probability of observing positive particles of a given spin and momentum is $2\pi(E'/L)dN_s/dk' = \langle b'^\dagger_s(k) b'_s(k) \rangle$.

GRADING:

This problem will be graded on a scale of 40. All correct $\rightarrow$ 40 pts, anything wrong $\rightarrow$ 0 pts, if units are wrong or rate does not go to zero for $M = 0 \rightarrow$ -40 pts. The class will turn in one copy of the solution at 3PM Monday.