Fine Art of global fitting and Error Estimates

Jon Pumplin – 29 Sept 2003

High energy hadrons interact through their quark and gluon constituents. The interactions become weak at short distances due to the asymptotic freedom property of Quantum Chromodynamics, allowing perturbation theory to be applied to a rich variety of experiments.

The nonperturbative nature of the proton for single interactions is characterized by Parton Distribution Functions $f(Q, x)$ of momentum scale $Q$ and light-cone momentum fraction $x$ for each flavor. Evolution in $Q$ is determined perturbatively by QCD renormalization group equations, so $f(Q, x)$ can be defined by functions $f(Q_0, x)$ of $x$ at a fixed small $Q_0$. Those functions are measured by fitting a wide range of data.

Known and unknown systematic errors pose a challenge to global fitting.
Outline of talk

- Introduction to PDFs
- Handling correlated experimental errors
- Estimating uncertainties
- Eigenvector PDF sets
- Lagrange multipliers
- Reweighting experiments
- Bootstrap methods
- Application: Jet predictions
- Application: Strangeness asymmetry and NuTeV anomaly

Global QCD analysis

- Extract universal non-perturbative features of proton or nucleus from large variety of experiments
  - Factorization
    (Short distance and long distance separable)
  - Asymptotic Freedom
    (Hard scattering perturbatively calculable)
  - Renormalization Group Evolution in scale $Q$
    (PDFs characterized by functions of $x$ at $Q_0$)
- Test consistency of QCD – overall and with individual experiments
- Make results available – needed by all experiments with hadron beams or targets: HERA, RHIC, Tevatron, LHC, non-accelerator
- Explore the range of uncertainties
Factorization Theorem

\[ F_A^\lambda(x, \frac{m}{Q}, \frac{M}{Q}) = \sum_a f_A^a(x, \frac{m}{\mu}) \otimes F_a^\lambda(x, \frac{Q}{\mu}, \frac{M}{Q}) + \mathcal{O}(\frac{\Lambda}{Q})^2 \]

Experimental Input

Parton Distributions:
- Nonperturbative parametrization at \( Q_0 \)
- DGLAP Evolution to \( Q \)

Hard Scattering:
(Perturbatively Calculable)
Kinematic region covered by data

Data with a wide range of scales are tied together by the DGLAP renormalization group evolution equation.

Consistency or inconsistency between the different processes can be observed only by applying QCD to tie them together in a global fit.

HERA II, Tevatron run II (W, Z production), and LHC will dramatically extend the range and accuracy.
CTEQ6 Global analysis

Input from Experiment:
- \( \sim 2000 \) data points with \( Q > 2 \) GeV from \( e, \mu, \nu \) DIS; lepton pair production (DY); lepton asymmetry in \( W \) production; high \( p_T \) inclusive jets; \( \alpha_s(M_Z) \) from LEP

Input from Theory:
- NLO QCD evolution and hard scattering
- Parametrize at \( Q_0 \):
  \[
  A_0 x^{A_1} (1 - x)^{A_2} e^{A_3x} (1 + A_4x)^{A_5}
  \]
- \( s = \bar{s} = 0.4 (\bar{u} + \bar{d})/2 \) at \( Q_0 \); no intrinsic \( b \) or \( c \)

Construct effective \( \chi^2_{\text{global}} = \sum_{\text{expts}} \chi^2_n \):
- \( \chi^2_{\text{global}} \) includes the known systematic errors
- Minimizing \( \chi^2_{\text{global}} \) yields “Best Fit” PDFs.
- Variation of \( \chi^2_{\text{global}} \) in neighborhood of the minimum defines uncertainty limits.
- Estimate uncertainty as region of parameter space where \( \chi^2 < \chi^2(\text{Best Fit}) + T^2 \) with \( T \approx 10. \)

(Quite different from Gaussian statistics because of unknown correlated systematic errors in theory and experiments – as measured by inconsistency between experiments).
• Valence quarks dominate for $x \to 1$
• Gluon dominates for $x \to 0$, especially at large $Q$
Comment on Parametrization

For $d_{\text{val}}$, $u_{\text{val}}$, or $g$, we use

$$xf(x, Q_0) = A_0 x^{A_1} (1 - x)^{A_2} e^{A_3 x} (1 + e^{A_4 x})^{A_5}$$

This corresponds to

$$\frac{d}{dx} \ln (xf) = \frac{A_1}{x} - \frac{A_2}{1 - x} + \frac{c_3 + c_4 x}{1 + c_5 x}$$

i.e., we add a 1:1 Padé form to the singular terms of the traditional $A_0 x^{A_1} (1 - x)^{A_2}$ parametrization.

A sufficiently flexible parametrization is important; but for convergence, there must not be too many “flat directions.” For that reason, some of the parameters are frozen for some flavors.

(To measure a set of continuous PDF functions at $Q_0$ on the basis of a finite set of data points would appear to be an ill-posed mathematical problem. However, this difficulty is not so severe as might be expected since the actual predictions of interest that are based on the PDFs are discrete quantities. In particular, fine-scale structure in $x$ in the PDFs at $Q_0$ tend to be smoothed out by evolution in $Q$. They correspond to flat directions in $\chi^2$ space, so they are not accurately measured; but they have little effect on the applications of interest.)
\[ \chi^2 \text{ and Systematic Errors} \]

The simplest definition

\[ \chi^2_0 = \sum_{i=1}^{N} \frac{(D_i - T_i)^2}{\sigma_i^2} \]

\[ \begin{cases} D_i = \text{data} \\ T_i = \text{theory} \\ \sigma_i = \text{“expt. error”} \end{cases} \]

is optimal for random Gaussian errors,

\[ D_i = T_i + \sigma_i r_i \quad \text{with} \quad P(r) = \frac{e^{-r^2/2}}{\sqrt{2\pi}}. \]

With systematic errors,

\[ D_i = T_i(a) + \alpha_i r_{\text{stat},i} + \sum_{k=1}^{K} r_k \beta_{ki}. \]

The fitting parameters are \( \{a_\lambda\} \) (theoretical model) and \( \{r_k\} \) (corrections for systematic errors).

Published experimental errors:

- \( \alpha_i \) is the ‘standard deviation’ of the random uncorrelated error.

- \( \beta_{ki} \) is the ‘standard deviation’ of the \( k \)th (completely correlated!) systematic error on \( D_i \).
To take into account the systematic errors, we define
\[
\chi^2(a_\lambda, r_k) = \sum_{i=1}^{N} \frac{(D_i - \sum_k r_k \beta_{ki} - T_i)^2}{\alpha_i^2} + \sum_k r_k^2,
\]
and minimize with respect to \( \{r_k\} \). The result is
\[
\hat{r}_k = \sum_{k'} (A^{-1})_{kk'} B_{k'}, \quad \text{(systematic shift)}
\]
where
\[
A_{kk'} = \delta_{kk'} + \sum_{i=1}^{N} \frac{\beta_{ki} \beta_{k'i}}{\alpha_i^2},
\]
\[
B_k = \sum_{i=1}^{N} \frac{\beta_{ki} (D_i - T_i)}{\alpha_i^2}.
\]
The \( \hat{r}_k \)'s depend on the PDF model parameters \( \{a_\lambda\} \). We can solve for them explicitly since the dependence is quadratic.

We then minimize the remaining \( \chi^2(a) \) with respect to the model parameters \( \{a_\lambda\} \).

- \( \{a_\lambda\} \) determine \( f_i(x, Q_0^2) \).
- \( \{\hat{r}_k\} \) are the optimal “corrections” for systematic errors; i.e., systematic shifts to be applied to the data points to bring the data from different experiments into compatibility, within the framework of the theoretical model.
Comparison of CTEQ6M fit to data sets with correlated systematic errors

<table>
<thead>
<tr>
<th>data set</th>
<th>$N_e$</th>
<th>$\chi^2_e$</th>
<th>$\chi^2_e/N_e$</th>
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<tr>
<td>BCDMS p</td>
<td>339</td>
<td>377.6</td>
<td>1.114</td>
</tr>
<tr>
<td>BCDMS d</td>
<td>251</td>
<td>279.7</td>
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</tr>
<tr>
<td>H1a</td>
<td>104</td>
<td>98.59</td>
<td>0.948</td>
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<tr>
<td>H1b</td>
<td>126</td>
<td>129.1</td>
<td>1.024</td>
</tr>
<tr>
<td>ZEUS</td>
<td>229</td>
<td>262.6</td>
<td>1.147</td>
</tr>
<tr>
<td>NMC F2p</td>
<td>201</td>
<td>304.9</td>
<td>1.517</td>
</tr>
<tr>
<td>NMC F2d/p</td>
<td>123</td>
<td>111.8</td>
<td>0.909</td>
</tr>
<tr>
<td>DØ jet</td>
<td>90</td>
<td>69.0</td>
<td>0.766</td>
</tr>
<tr>
<td>CDF jet</td>
<td>33</td>
<td>48.57</td>
<td>1.472</td>
</tr>
</tbody>
</table>

Observe that $\chi^2/N_{\text{pt}}$ is close to 1.0 — but not as close as would be expected if we lived in the idealized world of statistics.
CTEQ6M fit to ZEUS data at low $x$

The data points include the estimated corrections for systematic errors. That is to say, the central values plotted have been shifted by an amount that is consistent with the estimated systematic errors, where the systematic error parameters are determined using other experiments via the global fit.

The error bars are statistical errors only.
(a) Histogram of residuals for the ZEUS data. The curve is a Gaussian of width 1.

(b) Similar comparison without corrections for systematic errors on the data points.
Systematic shifts for the ZEUS data (10 systematic errors)

Systematic shifts for the NMC data (11 systematic errors)
These inclusive jet cross section measurements provided the first major stimulus to the study of PDF uncertainties — in particular, the uncertainties associated with choices made in the form of parametrizations at $Q_0$. 
Values of the fitted systematic error parameters for CDF Inclusive jet cross section:

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\hat{r}_k$</th>
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<tbody>
<tr>
<td>1</td>
<td>-0.511</td>
</tr>
<tr>
<td>2</td>
<td>0.816</td>
</tr>
<tr>
<td>3</td>
<td>0.022</td>
</tr>
<tr>
<td>4</td>
<td>1.347</td>
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<tr>
<td>5</td>
<td>-1.307</td>
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<tr>
<td>6</td>
<td>0.089</td>
</tr>
<tr>
<td>7</td>
<td>-0.222</td>
</tr>
</tbody>
</table>

All parameters are $\lesssim 1$ as they should be.
Sources of uncertainty:
1. Experimental errors included in $\chi^2$
2. Unknown experimental errors
3. Parametrization dependence
4. Higher-order corrections & Large Logarithms
5. Power Law corrections ("higher twist")

Fundamental difficulties:
1. Good experiments run until systematic errors dominate: the magnitude of remaining systematic errors involves guesswork.
2. Systematic errors of the theory and their correlations are even harder to guess.
3. Quasi–ill-posed problem: determine continuous functions from discrete data set
4. Some combinations of variables are unconstrained, e.g., $s - \bar{s}$ before NuTeV data.

Approach
Use "$\chi^2$" as measure of fit, but vary weights of experiments to estimate range of acceptable fits, rather that relying on the classical $\Delta \chi^2 = 1.$
Suppose the quantity $\theta$ is measured by two different experiments, or extracted using two different approximations to the True Theory.

What would you quote as the Best Fit and the Uncertainty?
MSU/CTEQ uncertainty methods

2-dim illustration of the neighborhood of the global minimum in the 16-dim parton parameter space ...

\( \chi^2 \)-contours

- **Hessian Matrix Method:** eigenvectors of error matrix yield 40 sets \( \{ S_i^{\pm} \} \) that are displaced “up” or “down” by \( \Delta \chi^2 = 100 \) from the best fit. Get error by sum of squares and construct extreme PDFs for any observable; or simply look at extremes from the 40 sets.

- **Lagrange Multiplier Method:** Track \( \chi^2 \) as function of \( F \) (e.g. \( \sigma_W \)) by minimizing \( \chi^2 + \lambda F \). Yields special-purpose PDFs that give extremes of \( \sigma_W \), or \( \langle y \rangle \) for rapidity distribution of \( W \), or \( \sigma \) for \( tt\bar{t} \) production; or \ldots
Hessian (Error Matrix) method

Classical error formulae

\[ \Delta \chi^2 = \sum_{ij} (a_i - a_i^{(0)})(H)_{ij}(a_j - a_j^{(0)}) \]

\[ (\Delta F)^2 = \Delta \chi^2 \sum_{ij} \frac{\partial F}{\partial a_i} (H^{-1})_{ij} \frac{\partial F}{\partial a_j} \]

Hessian matrix \( H \) is inverse of error matrix.

Direct application fails because of extreme differences in variation of \( \chi^2 \) for different directions in parameter space ("steep" and "flat" directions), as shown by large range of eigenvalues of \( H \):
Convergence problems in the minimization are solved by an iterative method that finds and rescales the eigenvectors of $H$, leading to a diagonal form

$$\Delta \chi^2 = \sum_{i} z_i^2$$

$$(\Delta F)^2 = \sum_{i} \left( F(S_i^{(+)}) - F(S_i^{(-)}) \right)^2$$

where $S_i^{(+)}$ and $S_i^{(+)}$ are PDF sets that are displaced along the eigenvector directions.

The eigenvector PDF sets are published, along with the Best Fit, for estimating PDF uncertainties of predictions.
New ways to measure consistency of fit
(Work in progress with John Collins)

Key idea: In addition to the

Hypothesis-testing criterion: $\Delta \chi^2 \sim \sqrt{2N}$

use the stronger

Parameter-fitting criterion: $\Delta \chi^2 \sim 1$

Parameters here are relative weights assigned to

various experiments, or to results obtained using

various experimental methods. Examples:

- Plot minimum $\chi_i^2$ vs. $\chi_{\text{tot}}^2 - \chi_i^2$, where $\chi_i^2$ is one
  of the experiments, or all data on nuclei, or all
data at low $Q^2$, ...

or

- Plot both as function of Lagrange multiplier $u$
  where $(1 - u)\chi_i^2 + (1 + u)(\chi_{\text{tot}}^2 - \chi_i^2)$ is the
  quantity minimized.

Can obtain quantitative results by fitting to a model

with a single common parameter $p$:

$$\chi_i^2 = A + \left(\frac{p}{\sin \theta}\right)^2 \Rightarrow p = 0 \pm \sin \theta$$

$$\chi_{\text{not-i}}^2 = B + \left(\frac{p-S}{\cos \theta}\right)^2 \Rightarrow p = S \pm \cos \theta$$

These differ by $S \pm 1$, i.e., by $S$ “standard deviations”
Fits to 8 of the experiments in the CTEQ5 analysis

<table>
<thead>
<tr>
<th>Expt</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>2.7</td>
<td>3.3</td>
<td>3.3</td>
<td>4.2</td>
<td>5.3</td>
<td>7.6</td>
<td>7.4</td>
<td>8.3</td>
</tr>
<tr>
<td>$\tan\phi$</td>
<td>0.56</td>
<td>0.54</td>
<td>0.99</td>
<td>0.86</td>
<td>0.71</td>
<td>1.14</td>
<td>0.65</td>
<td>0.39</td>
</tr>
</tbody>
</table>
Lessons from these reweighting studies

• Global analysis requires compromises – the PDF model that gives the best fit to one set of data does not give the best fit to others. This is not surprising because there are systematic differences between the experiments.

• The scale of acceptable changes of $\chi^2$ must be large. Adding a new data set and refitting may increase the $\chi^2$'s of other data sets by amounts $>> 1$. 
The question of tolerance

For the specified tolerance (\( \Delta \chi^2 = T^2 \)) there is a corresponding range of uncertainty, \( \pm \Delta X \).

What should we use for \( T \)?

\[ \chi^2_{\text{global}} \]

\[ \chi^2_{\text{min}} + \Delta \chi^2 \]

\[ \chi^2_{\text{min}} \]

\( X \): any variable that depends on PDF’s

\( X_0 \): the prediction in the standard set

\( \chi^2(X) \): curve of constrained fits

\( \Delta \chi^2 \)
Fractional uncertainty of gluon

Uncertainty bands (envelope of possible fits) for the gluon distribution at $Q^2 = 10 \text{ GeV}^2$.

Curves show CTEQ5M1 (solid), CTEQ5HJ (dashed), MRST2001 (dotted)

Differences between these are comparable to the estimated uncertainty(?!)

Uncertainties of quark distributions are much smaller than this because DIS measurements see the quark charge in leading order.
Statistical Bootstrap method

Generate random weights for each of the 16 experiments in global fit by \( \frac{dP}{dW_i} = e^{-W_i} \). Find best fit for each set of weights. Repeat 200 times and take the central 90% at each \( x \) as the measure of uncertainty range. Shows sizable uncertainty with no ad hoc assumption such as \( \Delta\chi^2 = 100 \).

Traditional statistical bootstrap (Efron and Tibshirani) uses integer weights 0 – 16 defined by random selection; this continuum method is similar but avoids zero weights.
Summary of Uncertainty Methods

Consistent estimates of the uncertainty ranges are found using several different methods:

- “Hessian Method” – eigenvectors of the error matrix
- “Lagrange Multiplier Method” – variation of $\chi^2$
- systematic reweighting of experiments
- random reweighting (statistical bootstrap)
Uncertainty of Gluon distribution

Red: Weight 50 for CDF Jet  Blue: Weight 50 for DØ Jet

Consistency check: Estimated uncertainty is comparable to the difference between nominally similar experiments.

Area under curve is proportional to momentum fraction carried by gluon – strongly constrained by DIS data. Hence the envelope itself is not an allowed solution.

Convergent Evolution: Uncertainty smaller at large $Q$
Application: $W$ rapidity distribution

Our methods allow us to calculate the extreme predictions due to PDF uncertainty for whatever quantity is of experimental interest.

For example, extremes of $\sigma_W$, $\langle y \rangle$, $\langle y^2 \rangle$ for $W$ production at FNAL – relevant for $M_W$ measurement:

Same curves after subtracting central values:

Important for measuring $W$ mass at FNAL.
Measurement of $\alpha_s$

We find that the CTEQ6 analysis is nicely consistent with the World Average determination of $\alpha_s(M_Z)$. But it is not precise enough to improve that value.
Application: Measurement of $\alpha_S(M_Z)$:

If assume $\Delta \chi^2 = 1$ criterion in each experiment, the experiments are inconsistent.

Our error estimate ($T = 10$) is

$$\alpha_S(M_Z) = 0.1165 \pm 0.0065$$

This corresponds to somewhat conservative assumptions – perhaps to be thought of as an effective “2 $\sigma$” limit. Hence it is comparable to the MRST limit based on $T = 5$. 
Application: Uncertainties of luminosity functions at LHC

- One component of the uncertainty in predicting the Higgs production cross section at LHC is an uncertainty of 8% due to PDF uncertainty.
Application: Inclusive jet ratio

Inclusive jet energy dependence

$$\frac{d\sigma}{dP_T}(1.96 \text{ TeV})$$

$$\frac{d\sigma}{dP_T}(1.80 \text{ TeV})$$

between Tevatron Run I and Run II offers a sensitive test of QCD and a probe for quark substructure, because many systematic errors cancel. Right now it is an important check on the experimental jet “energy scale” calibration.

Prediction and uncertainty range from CTEQ6.1
Inclusive jet production and the search for new physics

Inclusive jet cross section: D0 data and 40 alternate PDF sets

Fractional differences

(hep-ph/0303013)
Is there room for new physics from Run Ib?

Contact interaction model with $\Lambda = 1.6, 2.0, 2.4$ TeV
The inclusive jet cross section versus pT for 3 rapidity bins at the LHC. Predictions of all 40 eigenvector basis sets are superimposed.
Strangeness Asymmetry and NuTeV

Background:

- CCFR–NuTeV measurements of dimuon production in $\nu$, $\bar{\nu}$ scattering (on Fe) (2001)
- NuTeV measurement of the Weinberg angle via Paschos-Wolfenstein ratio (2002)

$$\frac{\sigma_{NC}^\nu - \sigma_{NC}^{\bar{\nu}}}{\sigma_{CC}^\nu - \sigma_{CC}^{\bar{\nu}}} \approx \frac{1}{2} - \sin^2 \theta_W$$

3.1 $\sigma$ discrepancy with world average:

$$\sin^2 \theta_W = 0.2277 \pm 0.0016 \quad [\text{NuTeV}]$$
$$\sin^2 \theta_W = 0.2227 \pm 0.0004 \quad [\text{LEP EWWG}]$$

Recent development: CTEQ Global analysis with $s(x) \neq \bar{s}(x)$, including the dimuon data

- Preliminary report at Lepton-Photon 2003: $s(x) > \bar{s}(x)$ at large $x$ may remove some or all of the anomaly.
- Much further work in progress: previous global fits assumed $s(x) = \bar{s}(x) = \kappa [\bar{u}(x) + \bar{d}(x)]$ at $Q_0$.
- More experiments sensitive to $s$, $\bar{s}$ would help.
Corrections to $R^{-}$

$$R^{-} = \frac{1}{2} - s^2_w \left( \delta N \frac{\int x(u_v - d_v) dx}{\int x(u_v + d_v) dx} + \frac{\int x(s - \bar{s}) dx}{\int x(u_v + d_v) dx} \right) \left[ 1 - \frac{7}{3} s^2_w + \frac{4\alpha_s}{9\pi} \left( \frac{1}{2} - s^2_w \right) \right]$$

Neutron excess
Strange asymmetry

These corrections have been under close scrutiny by many authors, in particular BPZ (Barone et al.) and Davidson et al.

Kulagin
hep-ph/0301045
Strangeness Structure of the Nucleon: Dimuon Production in Scattering

\( \nu, \bar{\nu} \)

# of events:

<table>
<thead>
<tr>
<th></th>
<th>NuTeV</th>
<th>CCFR</th>
<th>Combined</th>
</tr>
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<tbody>
<tr>
<td>Neutrino</td>
<td>5012</td>
<td>5030</td>
<td>10042</td>
</tr>
<tr>
<td>Anti-Nu</td>
<td>1458</td>
<td>1060</td>
<td>2518</td>
</tr>
</tbody>
</table>

*(d is Cabibbo suppressed)*

* High stats & high precision data
* Best constraints on strange quark

\[
\frac{d \sigma^+_{\mu^+\mu^-}}{dx \ dy} = \int d \Gamma d \Omega \frac{d \sigma^+_{\mu^+\mu^-}}{dx \ dy \ d \Gamma} \otimes D_c(\Gamma) \otimes \Delta_c(\Omega) \bigg|_{\mu^+ > 5 \ GeV}
\]

Di-muon cross-section
Charm Production cross-section
Fragmentation Function
Decay Distribution

Modeling needed to compare theory with data.

M. Goncharov et al., NuTeV Collaboration PRD 64:110226 (2001)
CCFR-NuTeV Analysis of Strange Quarks and the Weinberg Angle Measurement

• Ingredients to the CCFR-NuTeV dimuon analysis:
  – Data on $\nu N, \bar{\nu} N \rightarrow \mu^+ \mu^- + X$
  – Fragmentation functions
    Peterson, Schlatter, Schmitt, Zerwas '83; Collins, Spiller '85
  – Buras-Gaemer /CTEQ/GRV non-strange partons
  – Strange distributions assumed given by
    \[ s(x, Q^2) = \kappa_\nu \frac{\bar{u}(x, Q^2) + \bar{d}(x, Q^2)}{2} (1 - x)^{\alpha_\nu} \]
    \[ \bar{s}(x, Q^2) = \kappa_\bar{\nu} \frac{\bar{u}(x, Q^2) + \bar{d}(x, Q^2)}{2} (1 - x)^{\alpha_{\bar{\nu}}} \]

• $\Rightarrow$ Gave parameters $\kappa$ and $\alpha$, but no actual plots of $s(x, Q)$, …
For implication on NuTeV anomaly, the key is the Strangeness Asymmetry. Define:

\[ [s^\pm] \equiv \int_0^1 s^\pm(x) \, dx \equiv \int_0^1 [s(x) \pm \bar{s}(x)] \, dx \]

and the corresponding momentum fractions:

\[ [S^\pm] \equiv \int_0^1 S^\pm(x) \, dx \equiv \int_0^1 x[s(x) \pm \bar{s}(x)] \, dx \]

In particular, it is \([S^-]\) that enters the P-W ratio correction term.

CCFR-NuTeV claimed \([S^-] \sim -0.0027\) opposite to direction that would decrease the anomaly.
Almost the same ingredients as CTEQ6 analysis
Add CCFR-NuTeV dimuon data (and a few more)
Allow a non-symmetric strangeness sector:

Parametrization of the Strangeness sector (at some $Q=Q_0$)

\[
s^+(x, Q_0) = A_0 x^{A_1} (1-x)^{A_2} P_+(x; A_3, A_4, \ldots)
\]

\[
s^-(x, Q_0) = s^+ \tanh[a x^b (1-x)^c P_-(x; x_0, d, e, \ldots)]
\]

\[
P_-(x) = \left(1 - \frac{x}{x_0} + dx^2 + ex^3 + \ldots\right)
\]

Where $x_0$ is to be determined by the condition $[s-] = 0$. 
Preliminary Results

positive \([S^-]\) case

Negative \([S^-]\) case: flipped.
Central flaw of NuTeV analysis: “CCFR ansatz”:

\[
\begin{align*}
    s(x, Q^2) &= \kappa_v \frac{\bar{u}(x, Q^2) + \bar{d}(x, Q^2)}{2} (1 - x)^{\alpha_v} \\
    \bar{s}(x, Q^2) &= \kappa_v \frac{\bar{u}(x, Q^2) + \bar{d}(x, Q^2)}{2} (1 - x)^{\alpha_v}
\end{align*}
\]
Results on the strange sea asymmetry from BPZ

without CCFR, same asymmetry as in previous studies

\[ \int_0^1 (x\bar{s} - x\bar{s}) \, dx = 1.8 \pm 0.5 \times 10^{-3} \]

using all data sets, the asymmetry is strongly reduced

\[ \int_0^1 (x\bar{s} - x\bar{s}) \, dx = 1.8 \pm 3.8 \times 10^{-4} \]

momentum asymmetry is compatible with zero
Figure 2.
Correlation between $\chi^2$ values and $[S^-]$

Red: dimuon cross section

Blue: other data sensitive to $s$–$s_{\text{bar}}$ (F3)
Outlook – I

- Parton Distribution Functions are a necessary infrastructure for precision Standard Model studies and New Physics searches at hadron colliders and experiments using hadron targets.
- PDFs of the proton are increasingly well measured.
- Useful tools are in place to estimate the uncertainty of PDFs and to propagate those uncertainties to physical predictions. There is adequate agreement between various methods for estimating the uncertainty.
- The “Les Houches Accord” interface makes it easy to handle the large number of PDF solutions that are needed to characterize uncertainties. [hep-ph/0204316]
Outlook – II

• Improvements in the treatment of heavy quark effects are in progress, and together with neutrino experiments they will allow improved flavor differentiation.
• PDFs summarize fundamental nonperturbative physics of the proton – a challenge to be computed! (Moments of meson PDFs have been done on lattice.)
• Other non-perturbative methods, e.g. for $s(x) - \bar{s}(x)$?
• HERA and Fermilab run II data will provide the next major experimental steps forward, followed by LHC.
• Theoretical improvements such as resummation to use direct photon and W transverse momentum data will be useful.
• In view of possible isospin breaking, and the importance of nuclear shadowing & anti-shadowing effects, HERA measurements on deuterons would be highly welcome.