

Worksheet #2 – PHY102 (Spring 2011)

Formats and List operations (Vectors)

Formating

From the Mathematica toolbar try the following: “format” followed by “style.” The dropdown menu offers you lots of options. The default is “input,” and that is format in which you do calculations. However Mathematica also allows various fonts and styles of text input. In this problem set, you should include a text “cell” before each problem to identify the problem (e.g. Problem 1), and a text cell at the top of the page with your name and the number of the worksheet (e.g. “Worksheet 2”).

Last week we did derivatives and integrals using the full Mathematica commands. However many of these commands may be entered from “palettes”. To activate a palette, click on the Palettes menu on the toolbar, and choose one, e.g., Basic Math Assistant.

When you get into trouble

Sometimes you will try to use a variable in more than one way. This can confuse Mathematica. There are several ways to clear a variable a , for example

$a = .$ which clears a numerical assignment to a , and
 $Clear[a[x]]$ which clears a function assignment to $a[x]$.

If you want to remove all of your prior definitions you can use,

$Remove[“Global’ *”]$

At some point Mathematica will get really unhappy and start doing a really long-winded calculation which you did not think you asked it to do. In that case you can go to the “kernel” tab, and click on “abort evaluation”. Sometimes that does not work, in which case you can click on “quit kernel”. This stops the Mathematica kernel and you lose the evaluations you have already carried out. (It may take a little time to die; you can use that time to swear at it silently. If that doesn’t work, you can kill the Mathematica process that is running — if necessary by opening a terminal and running the unix command “top” to find the number of the process, and then using

the unix command “kill nnnn” where nnnn is the process number.)

Lists and Vectors

By now you must have read about vectors. A vector is a quantity which, unlike a scalar, can have many components. For example in Newton’s second law of motion

$$\vec{F} = m \frac{d^2 \vec{r}}{dt^2} \quad (1)$$

the quantity m (mass) is a scalar. But the force \vec{F} and the acceleration $\vec{a} = \frac{d^2 \vec{r}}{dt^2}$ are vectors. As you can see in Eq. (1), and which is true in general, multiplying a vector \vec{a} by a scalar m gives a vector \vec{F} . A vector is described by its components in a chosen coordinate system. For example a vector \vec{A} in cartesian coordinates is given by (or more precisely, represented by) $\vec{A} = (A_x, A_y, A_z)$.

In Mathematica, vectors are represented in the same way. The object is called a *list*, because it can be used for more general objects such as matrices and tensors. In this worksheet we only work with vectors.

Type “A = {Ax, Ay, Az}”. This means Mathematica associates the object A with the list {Ax, Ay, Az}. Now type “B = {Bx, By, Bz}”. Type “Dot[A,B]”. This will give the dot product $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$. (You can also get this by just typing $A \cdot B$.) As you know, this dot product is equal to $|A||B| \cos(\theta)$, where θ is the angle between the vectors \vec{A} and \vec{B} .

Likewise, the cross product of two vectors ($\vec{A} \times \vec{B}$) yields another vector $\vec{C} = \{A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x\}$. Type “Cross[A,B]” and verify that you indeed get the above expression in terms of the components of \vec{A} and \vec{B} . Unit vectors can be easily written with lists as: $\hat{x} = \{1,0,0\}$, $\hat{y} = \{0,1,0\}$, $\hat{z} = \{0,0,1\}$. Check by using Mathematica that $\hat{x} \cdot \hat{y} = \hat{y} \cdot \hat{z} = \hat{z} \cdot \hat{x} = 0$.

You can see that the elements in the list {Ax, Ay, Az} of the vector $\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$ are its x , y , and z components. How do we access the individual components from A? Easy! – just type “A[[2]]” and check that this gives A_y . How would you get Mathematica to print out the second element of the cross product “Cross[A,B]”?

Assignment #2.

Problem 1. Consider two vectors $\vec{A} = (\frac{\sqrt{3}}{2}, \frac{1}{2}, 0)$, and $\vec{B} = (\frac{1}{2}, \frac{\sqrt{3}}{2}, 0)$. Using Mathematica:

- (i) Check that they are both of unit magnitude.
- (ii) Find $\vec{A} \cdot \vec{B}$.
- (iii) Find the angle between these two vectors.
- (iv) Find the cross product of these two vectors.

Problem 2. Consider the unit vectors along x, y, and z directions: $\hat{x} = \{1,0,0\}$ $\hat{y} = \{0,1,0\}$ $\hat{z} = \{0,0,1\}$. Verify that $\hat{x} \times \hat{y} = \hat{z}$, $\hat{y} \times \hat{z} = \hat{x}$, $\hat{z} \times \hat{x} = \hat{y}$.

Problem 3. Verify for any three vectors \vec{A} , \vec{B} , and \vec{C} , that $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$.

Problem 4. The motion of a particle is given by $\vec{r}(t) = A(\hat{x} \cos(\omega t) + \hat{y} \sin(\omega t))$, where A and ω are constants. Find its velocity \vec{v} . Calculate $\vec{\Omega} \times \vec{r}$ where $\vec{\Omega} = (0, 0, \omega)$, and verify that $\vec{v} = \vec{\Omega} \times \vec{r}$. Do you recognise this motion? Plot the motion to confirm your intuition (use the help menu to look up how to use the command “ParametricPlot” for this problem—you will need to choose values for a and ω).