Worksheet #4 – PHY102 (Spring 2011)

Solving equations

Solving equations in Mathematica

Look up how to solve algebraic equations exactly(Solve) and numerically(NSolve). If you have a transcendental equation (e.b. $x = \sin(x)$) you need to use "FindRoot," which will involve telling Mathematica where to begin looking for the root—since there may be more than one of them.

In simple kinematics and simple applications of Newton's second law, the physics is often described by a second order linear differential equation. This may be solved analytically using DSolve, or numerically using NDSolve. We shall consider initial value problems in which it is necessary to specify the initial conditions. In Newton's second law, this is the initial position and velocity. An example is:

```
DSolve[{ x''[t] + 0.05 x'[t] +x[t] == 1,x'[0]==0,x[0]==2},x[t],t]
```

Note the double equals ("==") occurs in all of the "Solve, DSolve ..." functions. It is Mathematica's way of expressing a "Truth" statement. Use the Mathematica help index to look up DSolve and see some other examples.

Extracting what you want

This is a pretty confusing, but essential, part of the Mathematica syntax. The solutions are given as a list of substitution rules. First you have to choose the element of the output list that you want. Then you have to correctly use the substitution rule. For example,

```
s = Solve[x^2 == 16]
```

will give

```
s = {{-4},{4}}
```

If you want the cube of the second of these solutions, you can use

```
x^3 /. s[[2]]
```

Problems

Problem 1.

(i) Find and print the real root of the equation:

$$x^3 + 2x^2 + x = 1 \ .$$

Then print the cosine of that root. Also print the numerical value of that root, using the N[...] function.

(ii) Plot the two functions, x and $2\ \tanh(x)$, on the same graph (use Plot). Then find and print the largest real root of the equation.

$$x = 2 \tanh(x) \ .$$

(iii) Find the largest real solution to the equation $\sin(x) - x/100 = 0$. Hint: use the Plot function to see what you're up against, and to find an appropriate starting point.

Problem 2.

Set up the differential equation for the displacement $x(t)$ of a simple harmonic oscillator with mass $m = 1$ and angular frequency $\omega = 2$. Use Mathematica to solve this differential equation (DSolve) to find x as a function of t. Plot its kinetic energy as a function of time, given $x(0) = 5$, $v(0) = x'[0] = 0$. Now add damping to the equation, in the form $0.05x'(t)$. Repeat your calculation with this damping term. Plot over a time that includes at least 10 periods of the motion. Is this *underdamped* or *overdamped* motion? Put your answer (including explanation!) in a text cell (The distinction between underdamped and overdamped is based on whether or not the displacement oscillates. The borderline is called critically damped, which is the case where the system approaches equilibrium most rapidly.)

Problem 3.

A projectile is thrown from earth with initial speed u at an angle θ above the horizontal.

(i) Program Mathematica functions describing its equations of motion along the x and the y directions as a function of time.

(ii) Program Mathematica to "Solve" for the range of this projectile motion using these functions. At what angle to the horizontal is the range maximal?